A Parametrical Determination of the Influence Region of Holes in Electromagnetic Devices by the Compensation Theorem

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***Abstract*—In order to guarantee the requested accuracy in electromagnetic computations, suitable refinements are requested around geometrical discontinuities as holes, ports and other openings, due to their impact on the field smoothness. The width of refinement region depends on the shape and dimension of the openings. The classical Compensation Theorem can provide a useful support in evaluating the influence domain because it provides the specific effect of the discontinuities. The paper discusses a procedure to evaluate the influence domain of the holes in electromagnetic structures and assesses its performance by analyzing a number of cases relevant in fusion technology applications.**

***Index Terms*—Finite element methods, Adaptive mesh refinement, Numerical analysis, Fusion reactors.**

I. Introduction

In spite of the fast increase in the computer performances, electromagnetic analysis of actual devices requires a number of relevant approximations regarding, for instance, the materials constitutive relationships, as well as the geometry of the various parts composing the device [1]. On the other side, if accurate solutions are required in a specific part of the domain, the actual geometry, at least of the relevant parts, should be considered. In such cases, suitable selective refinements could lead to a reasonable trade-off between accuracy and computational burden. In such cases, the size of the region to be refined, e.g. the area around a hole is chosen typically on the basis of the expertise of the researchers, with a time consuming trial and error approach.

This paper sketches a procedure to evaluate the Influence Domain (ID) of apertures in conducting structures on electromagnetic analyses. In order to gain general indications, the quite general classes of rectangular and elliptical shaped holes are considered. Then, to assess the effectiveness of the procedure, an extensive parametric analysis is proposed in the specific field of electromagnetic devices for Thermonuclear Fusion [2].

A reliable help in looking for the holes ID in linear electromagnetic systems could come from a suitable exploitation of the classical Compensation Theorem (CT). As a matter of fact the CT, initially proposed for linear circuits, can be generalized for electromagnetic field applications to provide the effect of a resistance variation in known linear electromagnetic system.

II. Mathematical Formulation

The CT is a well known property of linear circuits: it allows to effectively determinate the current or voltage variation along any branch of a linear a-dynamic circuit, due to a resistance or conductance variation in any other branch. The application of CT is particularly advantageous in the case of parametric analyses, since it requires solving a modified circuit, with a single *compensation* source, localized in the perturbed branch.

The CT can be easily extended to the analysis of static or quasi-static fields in linear electromagnetic systems, in the case of variations in conductibility profile. Approaches based on the CT theorem result particularly effective for parametrical and sensitivity analyses of linear electromagnetic devices. The extension of CT to electromagnetic fields has been proposed in many areas, e.g. for the optimal design [3] or Eddy Current Testing applications [4], [5].

The use of CT is proposed here for determining the ID of rectangular and elliptical shaped holes in linear conductive superficial electromagnetic structures Ω. As usual in the approaches based on the CT theorem, let's consider the following three linear a-dynamic problems:

- The first problem describes the current map $\mathbf{J_0(r)}$ in the nominal system, driven by the actual sources S and with the assigned Boundary Conditions (BC) on $\partial\Omega$;
- The second one describes the *perturbed* current map $\mathbf{J_0(r)} + \Delta\mathbf{J_0(r)}$ in the same system, but with a perturbation in the conductivity distribution $\Delta\sigma$ in a part Ω_D of the structure;
- Finally, the third one describes the current in the same system, but driven *only* by the compensation current source $\mathbf{J}_C(\mathbf{r}) = -\mathbf{J_0(r)}\,\Delta\sigma/\sigma$ in Ω_D, and vanishing BC on $\partial\Omega$.

It is quite easy to demonstrate that the second problem is the superposition of the first and the third ones. In this way, any variation in the conductance of the first problem can be evaluated by solving the third problem, characterized just by a compensation source and with homogeneous BC. Of course, in the particular case of a hole, $\Delta\sigma = \sigma$, and $\mathbf{J}_C(\mathbf{r}) = -\mathbf{J_0(r)}$.

The CT is well suited to estimate the ID of a hole; as a matter of fact, the ID can be estimated as *the region where the current pattern of the compensation problem (the third one) is significant*.

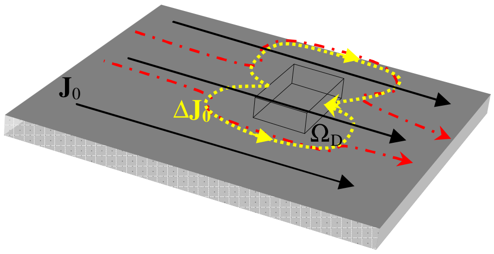

Fig. 1. Sketch of conducting domain Ω and of the hole region Ω_D

To guarantee the right flexibility, a Finite Element formulation is used here in the numerical computation of ID. In the present paper the numerical solutions are computed by the 3D integral code CARIDDI [6], [7], well assessed to face with the time-domain Maxwell equations in the magneto-quasi-stationary limit. In CARIDDI the electric vector potential **T** is adopted as unknown in such a way to automatically impose the solenoidality of the current density field, **J**, and the continuity of its normal component. Moreover, by using the edge elements for the numerical expansion of the electric vector potential it's possible to assume a numerical gauge based on the tree–cotree decomposition of the edges of the mesh retaining only the last ones as discrete unknowns. The final discretized model to be solved looks like the one of an Ohmic-inductive network $\underline{\underline{L}}\frac{d\underline{I}}{dt}+\underline{\underline{R}}\,\underline{I}=\underline{V_C}$ where:

$$R_{i,j}=\int_{\Omega_D}\eta\mathbf{w}_i\cdot\mathbf{w}_j\,d\Omega\,;$$

$$L_{i,j}=\frac{\mu_0}{4\pi}\int_{\Omega_D}\int_{\Omega_D}\frac{\mathbf{w}_i(\mathbf{r}')\cdot\mathbf{w}_j(\mathbf{r})}{|\mathbf{r}-\mathbf{r}'|}d\Omega'\,d\Omega \qquad (1)$$

$$V_C=-\int_{\Omega_s}\frac{\partial\mathbf{A}_C}{\partial t}\cdot\mathbf{w}_i\,d\Omega\,;\ \mathbf{A}_C=\frac{\mu_0}{4\pi}\int_{\Omega_s}\frac{\mathbf{J}_C(\mathbf{r}',t)}{|\mathbf{r}-\mathbf{r}'|}d\Omega'$$

and $\mathbf{J}_C$ is the impressed (compensation) current density localized in the source domain Ω_D. Note that $\underline{\underline{R}}$ is a symmetrical sparse matrix, whose elements $R_{i,j}$ do not vanish only if the i-th and j-th unknowns share the same mesh element. On the other hand $\underline{\underline{L}}$ is a symmetric full matrix because the $L_{i,j}$ coefficients keep in account the long-distance interactions between the unknowns.

III. EXAMPLE OF APPLICATION

Here, in order to assess the effectiveness of the numerical procedure, a simplified representation of a sector of the ITER Vacuum Vessel (VV) [2] has been considered. In the first problem (nominal configuration) the VV is assumed uniform, while in the second and third problem a square hole is considered. In the "nominal" configuration, eddy currents are induced in the VV passive structure by time varying currents in poloidal field coils. Simplified waveforms are considered here for the sake of illustration: each current ramp up to values in the range (-40÷40) kA in 1 s, and then remain constant, to simulate the basic characteristics behavior of current ramp-ups in an actual ITER shot.

The current density computed in the nominal configuration was used to estimate the "compensation current" into a square hole with an edge of about 7.6 cm in the upper part of a sector of the VV, and the compensation problem was solved until steady state was achieved. The map of "compensation" currents in the VV was then used to estimate the ID of the hole, by computing the norm of the current density, normalized to the current at hole edge, and drawing its contour lines using axis "normalized" to hole size. As an example, if an accuracy of 10% is sufficient in the resolution of the perturbed problem, than a refinement in a range about 2.5 times larger than the hole size is sufficient. (Fig. 2)

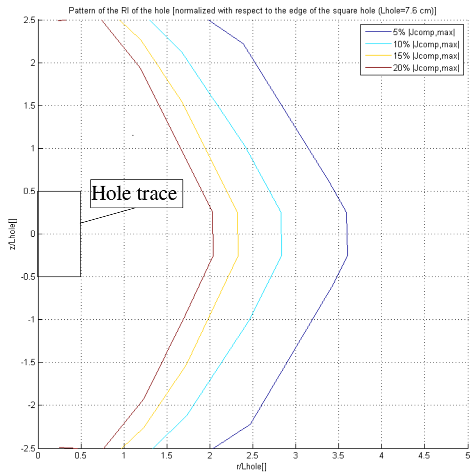

Fig. 2. Contour lines of the normalized norm (with respect to $|\mathbf{J}_c|$) of the current density generated by the compensation source

IV. OUTLOOK

In the full paper the procedure to determine the ID will be applied to rectangular and elliptical shaped holes, with varying sizes and aspect ratios. Different nominal current maps will also be considered, to assess the impact of hole alignment with respect to nominal current map. The procedure will be applied to both steady state and time-varying cases.

V. ACKNOWLEDGEMENTS

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