Numerical Modeling of Hysteresis in Si-Fe Steels

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***Abstract*— In this paper we discuss the numerical modeling of hysteresis in magnetic steels with oriented and non oriented grain. The modeling is fully 3-D, but it is applied here to typical 2-D magnetic problems. Starting from a vector generalization of the Preisach model, we introduce here the concept of an operative magnetic field, function of the state of magnetization of the material. This operative field is introduced in order to reproduce with more accuracy the peculiar behavior of Si-Fe steels approaching to the saturation state. The properties of the model are discussed and some examples are reported.**

***Index Terms*— Magnetic hysteresis, Oriented grain magnetic steels, Non Oriented grain magnetic steels, Preisach vector modeling.**

I. List of Symbols and Units

TABLE I
SYMBOLS AND UNITS

$\boldsymbol{H}_{ext}$	Applied (external) magnetic field (A/m).
$\boldsymbol{H}_{I}$	Interaction magnetic field (A/m).
$\boldsymbol{H}_{I0}$	Interaction magnetic field at zero magnetization (A/m).
$\boldsymbol{H}_{eq}$	Operative (equivalent) magnetic field (A/m).
$\boldsymbol{M}$	Magnetization (A/m).
$\boldsymbol{M}_{S}$	Magnetization at saturation state (A/m).
$\boldsymbol{h}_{ext}$	Per unit (relative) applied magnetic field (dimensionless).
$\boldsymbol{h}_{I}$	Per unit (relative) interaction magnetic field (dimensionless).
$\boldsymbol{h}_{I0}$	Per unit (relative) interaction magnetic field at zero magnetization (dimensionless).
$\boldsymbol{h}_{eq}$	Per unit (relative) operative magnetic field(dimensionless).
$\boldsymbol{m}$	Per unit (relative) magnetization (dimensionless).
μ_0	Magnetic permeability (H/m).
$\boldsymbol{B}$	Magnetic induction (T).
Ω	Material parameter vector.
$\boldsymbol{H}_{S}$	Hysteron switching field (A/m).
$\boldsymbol{h}_{S}$	Per unit (relative)hysteron switching field (dimensionless).

II. Introduction

The complete modeling of magnetic hysteresis for magnetic materials at macroscopic scale is a difficult task being the material behavior influenced by different factors such as the grain size, the structural stress, the presence of enclosures, etc. [1][2]. It is almost difficult to reproduce with accuracy the magnetic hysteresis and the static losses for several magnetic materials, in particular for the oriented grain OG Si-Fe magnetic steels. The use of phenomenological models [3] has shown several successful attempts to reproduce the macroscopic magnetic hysteresis. To this aim, one basic issue is the identification of the approximation functions to use in the model. A recent, promising approach to treat the magnetic vector hysteresis is an extension of the Classical Scalar Preisach Model from the 1-d case to the 3-d one [4][5]. The model is based on the definition of a vector hysteron characterized by a material-dependent Preisach distribution in the $\boldsymbol{H}$-space. In this paper we discuss the introduction of a operative magnetic field in order to reproduce with accuracy the magnetic behavior of OG and non oriented grain NOG Si-Fe Steels.

III. The Vector Modelling Of Hysteresis

We start this section by considering the vector extension of the Classical Scalar Preisach Model presented in a previous paper [6]. The model is based on the definition of a vector mathematical operator, also called vector hysteron, described in the $\boldsymbol{H}_{ext}$- space by a closed and convex critical surface. Each vector hysteron has a unique critical surface, described by a suitable set of parameters, indicated here as the vector Ω. As an example, we can assume a critical spherical surface: in this case the components of the vector Ω, are the coordinates of the center of the sphere H_{Ix}, H_{Iy}, and H_{Ix} and the value of its radius H_S. This example is represented in Fig. 1.

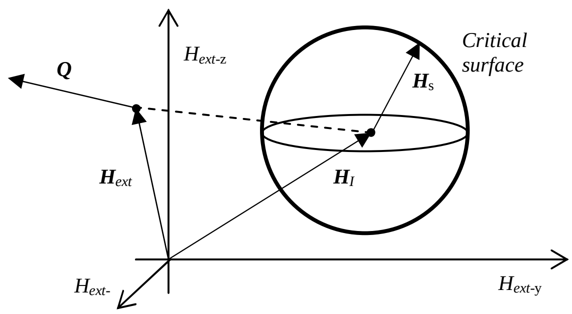

Fig. 1 - Working principle of the vector model.

The distance of the center of the critical surface from the origin is the interaction field H_I, that represents the global effect of thermal effects, stress effects and the presence of the other hysterons. Although this approach is phenomenological, the model is inspired to the static analysis of magnetic materials at micro-magnetic scale, where the fluctuation field, the magnetostatic and exchange field are properly defined [1]-[3]. The value of the hysteron radius H_S is function of the switching field, and it is function of the coercive field of the material. The magnetization state vector of the hysteron can be denoted by the unitary and dimensionless vector $\boldsymbol{Q}$. For applied magnetic fields inside the critical surface $\boldsymbol{Q}$ is frozen in the direction that it had just before it entered the critical surface, and it remains constant until it exits the critical surface. When exiting the critical surface $\boldsymbol{Q}$ instantly rotates so as to align itself along a new direction, perpendicular to the critical surface. This behavior is corresponding to the energy jump that occurs in the magnetic materials. Therefore for each hysteron and each value of the applied field $\boldsymbol{Q}$ is in general a multi-value function of Ω and $\boldsymbol{H}_{ext}$. The hysterons are suitably distributed in the $\boldsymbol{H}_{ext\text{-}plane}$, and their density distribution can be described by a dimensionless function P(Ω), defined in analogy with the classical Preisach model [3]. Each material, for a given thermal and mechanic stress condition, is defined and identified by a proper function P(Ω). The total magnetization is the vector sum of the magnetization due to all the vector hysterons.

IV. The Operative Magnetic Field

The basic idea introduced for the numerical modeling of hysteresis in a bulk material is the following definition of a so called operative, or equivalent magnetic field $\boldsymbol{H}_{eq}$

$$H_{eq} = H_{ext} - H_I \quad (1)$$

The hysterons, referred to a $\boldsymbol{H}_{eq}$- frame, are all centered in the origin. In addition, the interaction field is written as

$$H_I = H_{I0} + H_{IM} \quad (2)$$

where $\boldsymbol{H}_{I0}$ is the value of the interaction field at zero magnetization and is function of the thermal and mechanical stresses of the material, and $\boldsymbol{H}_{IM}$ is the change in interaction field due to the change in magnetization. Finally the magnetic field and the magnetization can be expressed in per unit, simply by dividing their values by the value of the magnetization at the saturation state, namely $\boldsymbol{M}_{S.}$ The model can be represented geometrically in Fig. 2.

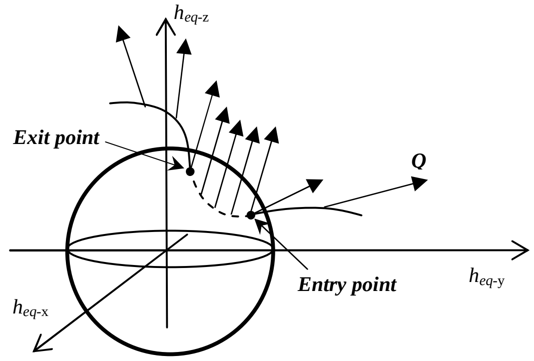

Fig. 2 - Representation of the vector model in the operative field reference frame.

The change of the interaction field due in function of the magnetization has been approximated here as follows

$$h_{IM} = k\, m \quad (3)$$

where *k* is a constant, to be determined experimentally.

V. Preliminary Results

We postulate now that the distribution function *P* is the product of three Lorentian's functions L_{HIx}, L_{HIy} and L_u

$$P = L_x \cdot L_y \cdot L_S \quad (4)$$

$$L_x = \frac{\sigma_x}{\sigma_x^2 + H_{I0x}^2} \quad L_y = \frac{\sigma_y}{\sigma_y^2 + H_{I0y}^2} \quad L_u = \frac{\sigma_S}{\sigma_S^2 + (H_S - H_C)^2} \quad (5)$$

where σ_x, σ_y, and σ_S are the shape parameters of each Lorentzian function and H_C is introduced in order to allow the peak of the Lorentzian distribution of the hysteron radius to move toward the value of the coercive field of the material.

The identification of the model parameters is a ill posed problem and we can have multiple solutions. The scope is to find a vector

$$\Phi = (\sigma_x \;\; \sigma_y \;\; \sigma_S \;\; H_C) \quad (6)$$

that minimizes the following fitness function *J* that defines the displacement between measured and computed data

$$J(\Phi) = \frac{1}{n}\sum_{i=1}^{n}[E(i) - C(\Phi, i)]^2 \quad (7)$$

where *i* is the i^{th} time sample, *n* is the number of all samples, *E(i)* is the i^{th} measured data, *C(i)* is the i^{th} computed data. The experimental data were measured in our laboratory using a Round Rotational Single Sheet Tester RRSST [7]. We have used in this case a genetic algorithm [8] for the model parameter identification. In Fig. 3 we show some preliminary results about numerical computation of magnetization of both NOG and OG samples. The experimental data are measured using a suitable set-up with a feed-back system to obtain a circular magnetization.

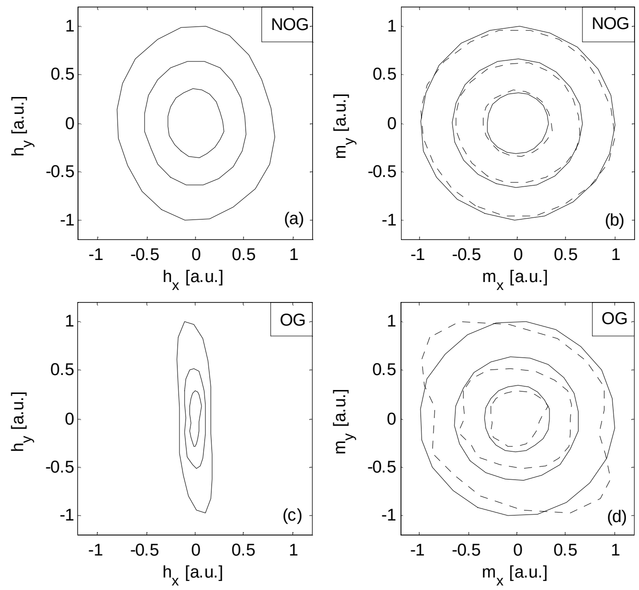

Fig. 3 - Comparison between measured (solid line) and computed (dotted line). The measured magnetic field is used as input for the hysteresis model.

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