

Voxel Based Finite Element Method Using Homogenization

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Abstract — The voxel mesh is one of the effective technique for reducing the cost of mesh generation. However poor ability of expression for curved surfaces is a drawback to be solved and decreases the accuracy of finite element analyses. In this paper, we introduce a homogenization method for the voxel elements on the surfaces of magnetic objects. The anisotropic permeability determined by the present method improves the accuracy of analyses. The numerical results show that the proposed method has smaller errors of inductance in a three dimensional magnetostatic analysis in comparison with the conventional method.

I. INTRODUCTION

The finite element (FE) analysis is known as a useful method for electromagnetic field analyses because of its high representation ability of model shapes. However, the cost of mesh generation is expensive and is often higher than that in the solver. Moreover, distorted elements, which are generated so as to fit the elements to the complex shape of objects, cause the slow convergence of linear equations to be solved.

In optimization for development of electromagnetic devices, a large number of FE analyses should be performed to evaluate each different shape of device [1]. In such cases, the fast mesh generation as well as fast convergence of solution are required. The voxel mesh could solve this difficulty. The shape of magnetic objects can be changed easily by remapping the permeability of each voxel without changing the structure of elements and nodes. Moreover, the voxel elements made of orthogonal grid yield the good convergence of the system equations. However, the voxel mesh approach has two serious weak points. The first one is that a large number of voxel elements are required to fill the air region. The second one is a poor ability of expression for curved surfaces: The terraced surfaces deteriorate accuracy of FE analyses. The former problem can be settled by introducing some numerical techniques such as non-conforming mesh [2] or the progress of computer resources. In the latter problem, spheres, for example, with small different radii are represented by the same shape expressed by voxel mesh. Moreover, discontinuous changes of shape are happened by the small variation of the radius. This causes a difficulty to evaluate the fitness value and sensitivity of fitness with respect to design parameters in parameter optimizations.

In order to overcome this difficulty, we introduce a homogenization method to determine the permeability of surface elements. The homogeneous permeability is calculated so as to fit the magnetic energy in the surface elements and

has different values for normal and tangential components of flux [3]-[5]. This technique results in a high accuracy of inductance computed with the conventional voxel FE analysis.

II. METHODS

Let us consider a magnetostatic field analysis using the FE method with voxel mesh. For simplicity, we consider mapping of a circle magnetic object with radius R to the voxel mesh shown in Fig. 1. The simplest method for mapping of permeability μ in each voxel is:

$$\mu = \begin{cases} \mu_m & \text{if } r_g < R \\ \mu_0 & \text{otherwise} \end{cases}, \quad (1)$$

where μ_m is the permeability of magnetic material, μ_0 is that in vacuum, and r_g is the distance from center of circle to the center of the element. This conventional method leads to disadvantages such as discontinuous changes of shape with small variation of the radius. To improve the treatment of surface, we introduce a volume (area) ratio γ in each voxel element obtained by,

$$\gamma = V_{in} / V_{all}, \quad (2)$$

where V_{in} is the volume of part such as $r_g < R$ and V_{all} is the volume of voxel element. Fig. 1(b) shows the distribution of γ . Moreover, let us consider determination of the homogeneous permeability μ' for each voxel element shown in Fig. 2, where μ' is determined in order that the magnetic energy in the element equal to that in the elements filled with the homogeneous material with μ' .

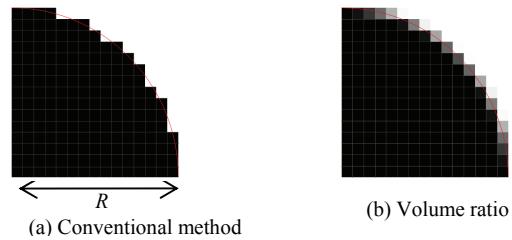


Fig. 1. Example of mapping of permeability in circle object.
1/4 model, $R=30\text{mm}$, size of voxel is $0.2 \times 0.2\text{mm}$.

11. Numerical Techniques

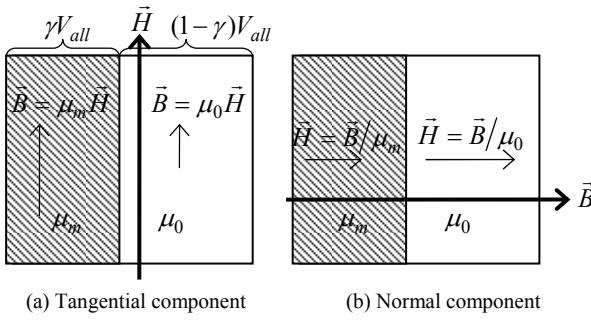


Fig. 2. homogenization method.

The magnetic energy ε for the tangential component of flux density \vec{B} and magnetic field \vec{H} are,

$$2\varepsilon = \int_V \vec{B} \cdot \vec{H} dV = \{\gamma\mu_m |\vec{H}|^2 + (1-\gamma)\mu_0 |\vec{H}|^2\} V_{all} . \quad (3)$$

$$= \mu'_t |\vec{H}|^2 V_{all}$$

where μ'_t is the homogeneous permeability corresponding to the tangential component of \vec{B} and \vec{H} . Equation (3) leads to,

$$\mu'_t = \gamma\mu_m + (1-\gamma)\mu_0 . \quad (4)$$

Similarly, the following relationships are satisfied for the normal component of \vec{B} and \vec{H} ,

$$\{\gamma |\vec{B}|^2 / \mu_m + (1-\gamma) |\vec{B}|^2 / \mu_0\} V_{all} = |\vec{B}|^2 V_{all} / \mu'_n , \quad (5)$$

$$\mu'_n = \frac{\mu_m \mu_0}{\gamma_n \mu_0 + (1-\gamma) \mu_m} . \quad (6)$$

From the above equations, the homogeneous permeability of the surface element with the normal unit vector $\vec{n} = (n_x, n_y)$ can be defined by,

$$\mu' = \begin{pmatrix} n_x \mu'_n + n_y \mu'_t & 0 \\ 0 & n_y \mu'_n + n_x \mu'_t \end{pmatrix} . \quad (7)$$

III. NUMERICAL RESULTS

To investigate the accuracy of the above described methods, we analyzed a 3D inductor model, which consists of a cylindrical iron and a coil, shown in Fig. 3. The region $0 \leq x, y, z \leq 0.1(m)$ is divided by 0.25mm size voxel elements (cubic elements). The other air region is divided by coarser elements. The inductance of this model is computed using magnetostatic FE analysis.

Fig. 4 shows the variation of inductance with increasing the radius R , where the conventional method is based on (1), μ' is given by (4) or (6) in the method A and B respectively, the proposed method is based on (7). We can see that the method B and the proposed method have good linear changes, while the conventional method and method A result in stepwise increase with R . Finally, we investigated the error of inductance by comparing with a highly accurate solution obtained by a FE analysis with very fine mesh using 1091245

tetrahedral elements. Table I indicates that the proposed method has good accuracy.

The formulation for non-linear permeability will be given in the full version of paper.

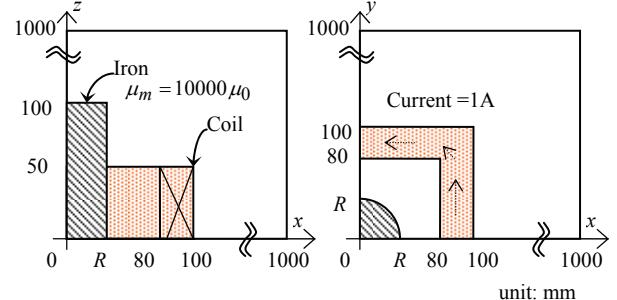


Fig. 3. Inductor model.

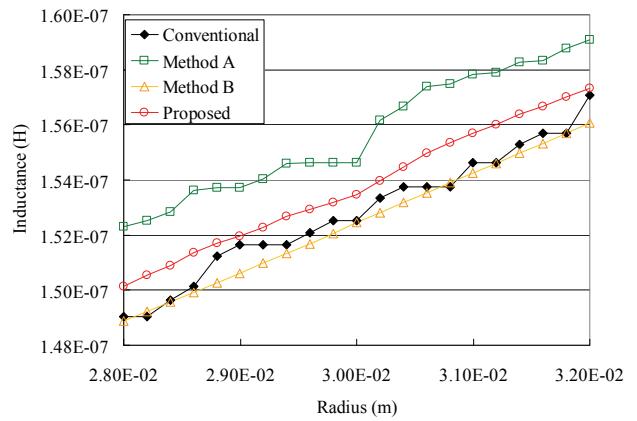


Fig. 4. Variation of Inductance with respect to radius.

TABLE I ERROR OF INDUCTANCE

Conventional	Method A	Method B	Proposed
0.484%	0.884%	0.543%	0.113%

$R = 30(\text{mm})$

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