Analytical Prediction of Cogging Torque for Spoke Type
Permanent Magnet Machines

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Abstract — A generalized analytical field solution by conformal transformation is presented for calculation of no-load magnetic field distribution in the slot-less and slotted air gap of spoke-type permanent magnet (PM) machines. The analytical field solutions are used to predict the cogging torque of an 8-pole, 48-slot spoke-type PM motor. The computed cogging torque using this method is compared with the 2D finite element analysis (FEA) results.

I. INTRODUCTION

Cogging torque in permanent magnet (PM) motors results from the interaction of the rotor permanent magnets and the stator slots, and can be reduced by optimizing the PM and the slot opening geometry parameters [1]. Although accurate calculations of cogging torque can be carried out using the finite element analysis (FEA), numeric methods are in general more time consuming. Alternatively, analytical field solutions [2]-[5] can perform the prediction of the cogging torque in a fraction of a second, and therefore are more efficient for the optimization process. However, all these approaches are suitable for surface PM motors only.

This paper presents a generalized analytical field solution using the conformal transformation [5]. This generalized analytical solution can be used for computation of the field distribution in the slot-less air gap, as well as the permeance distribution in the slotted air gap of spoke-type PM machines [6]. The cogging torque is derived directly from the derivative of the air-gap permeance with respect to the rotor position, instead of from the numeric derivative of the air-gap co-energy.

II. ANALYTICAL FIELD SOLUTIONS IN THE AIR GAP

In order to obtain generalized field solution for flux density and permeance distributions, one pole-pitch field region is considered, as shown in Fig. 1(a), which can be simplified to the region as shown in Fig. 1(b).

The field region in the \( z' \) plane of Fig. 1(b) can be transformed to the region in the \( z \) plane, as shown in Fig. 2, by the following transformation

\[
z = -j \cdot r_a \cdot (\ln(z') - \ln(r_a)) \tag{1}
\]

In Fig. 2, parameter \( 2b \) is the PM thickness, \( t_1 \) is the pole pitch, and \( g \) is the air-gap length.

The field region in the \( z \) plane of Fig. 2 can be transformed to an upper half of the \( w \) plane [5] by the Schwarz-Christoffel transformation

\[
z = \frac{2b}{\pi} \cdot \left( \arctan \left( \frac{2}{c} \cdot \frac{1+s}{1-s} \right) \right) \tag{2}
\]

where

\[
\begin{align*}
\begin{cases}
    s = \frac{c \cdot w}{\sqrt{1+c^2-w^2}} \\
    c = g / b
\end{cases}
\end{align*} \tag{3}
\]

In the field region of Fig. 2, assume that the magnetic potential on one rotor pole surface (from \( z_1 \) to \( z_2 \) to \( z_3 \)) is assigned to be \( -\phi_{m0} \) that on the stator surface (from \( z_4 \) to \( z_5 \) to \( z_6 \)) is assigned to be 0, and that on the other rotor pole surface (from \( z_7 \) to \( z_8 \) to \( z_9 \)) is assigned to be \( \phi_{m0} \). Then, the radial flux density can be derived as the function of \( \theta \)

\[
B_{mr}(\theta) = \mu_0 \cdot \frac{r_a}{r_c} \cdot \frac{\phi_{m0}}{g} \cdot n_B(\theta) \tag{4}
\]

where

\[
n_B(\theta) = \text{Re} \left( \frac{cw}{\sqrt{a^2-w^2}} \right) \tag{5}
\]

is the normalized air-gap radial flux density distribution. In (4), \( r_c \) is the radius of the center air gap, \( 0 \leq \theta \leq 90^\circ \) (electric degrees) corresponds to \( 0 \leq \phi \leq 0.5(\pi+\pi j) \) at the center air gap. The variable \( w \) in (5), as a function of \( \theta \), can be solved based on (2)-(3). When \( \theta > 90^\circ \), \( n_B(\theta) \) can be extended based on the symmetric conditions, as shown in Fig. 3.

The analytical solution (2)-(3) can be also used to calculate the air-gap permeance distribution. In such a case, the surface (from \( z_4 \) to \( z_5 \), \( z_1 \), \( z_2 \), \( z_3 \)) in Fig. 2 is the slotted
stator surface, and the surface (from \(z_3\) to \(z_4\)) is the smooth rotor surface, \(2b\) represents the slot opening width, and \(l_1\) stands for the slot pitch. Assume that the magnetic potential on the rotor surface (from \(z_3\) to \(z_4\)) is assigned to be \(\phi_{adv}\) and that on all other surface is assigned to be 0. The radial component of the flux density can be expressed as a function of \(\theta\)

\[
B_{r\theta}(\theta) = \mu_0 \cdot \frac{r_a}{r_c} \cdot \frac{\varphi_{ad\theta}}{g} \cdot \lambda_R(\theta)
\]

where

\[
\lambda_R(\theta) = \text{Re} \left( \frac{c}{\sqrt{a^2 - w^2}} \right)
\]

is the relative air-gap permeance, as shown in Fig. 4.

III. COGGING TORQUE COMPUTATION

The normalized air-gap flux density distribution in Fig. 3 can be expressed as \(n_d(\theta)\), where \(\theta\) is the angle in the rotating coordinate system, and the relative air-gap permeance distribution in Fig. 4 can be denoted as \(\lambda_R(\theta)\), where \(\theta_s\) is the angle in the static coordinate system. The angle difference between the two coordinate systems is defined as \(\theta = \theta_s - \theta_c\). Then the real air-gap flux density distribution is

\[
B_g(\theta, \theta_s) = \mu_0 \cdot \frac{r_a}{r_c} \cdot \frac{\varphi_{ad\theta}}{g} \cdot n_d(\theta) \cdot \lambda_R(\theta_s, \theta) + \theta
\]

The flux in one pole

\[
\phi(\theta) = l_a \cdot r_c \cdot \int_{0}^{2\pi/p} B_g(\theta, \theta_s) \cdot d\theta_s = \varphi_{mo} \cdot \Lambda_g(\theta)
\]

where

\[
\Lambda_g(\theta) = \mu_0 \cdot \frac{l_a r_a}{g} \cdot \int_{0}^{2\pi/p} n_d(\theta) \cdot \lambda_R(\theta_s, \theta) \cdot d\theta_s
\]

The flux per pole can be computed from the magnetic circuit with half-pole flux paths, and is given below

\[
\phi(\theta) = \frac{F_m}{R_g(\theta) + R_c + k_g R_m}
\]

where \(F_m\) and \(R_m\) are the half-thickness MMF and magnetic resistance of a permanent magnet, respectively, \(R_c\) is the magnetic resistances of the stator and rotor cores, \(k_g > 1\) is the leakage flux coefficient for the permanent magnet, and

\[
R_g(\theta) = 1/A_g(\theta)
\]

The magnetic co-energy in the motor is

\[
W(\theta) = \frac{P}{2} \cdot \frac{F_m \cdot \phi(\theta)}{R_g(\theta) + R_c + k_g R_m}
\]