Problem 6
Sphere in Uniform Magnetic Field

1. General Description

The problem to be addressed is that of a hollow sphere in a uniform sinusoidally varying magnetic field. Since the problem has an analytical solution, it is a convenient problem on which codes can be compared. The solution consists of the calculation of the magnetic field and eddy currents, as well as global quantities such as losses, stored energy, etc.

2. Mesh and Geometry Description

2a. Specific Mesh. In order to facilitate comparison in a meaningful way, a mesh is provided. This mesh should be generated as close as possible to the data given here. Because of the nature of the problem, most 3-D mesh generators should have little problem in generating this mesh. It can also be generated directly without the use of a mesh generator. The data given here has been specifically designed to simplify the reproduction of the mesh. All nodes of the mesh lie either on parallel planes or on concentric spheres. Only eight-node bricks are used and these have been distorted as little as possible. One eighth of the sphere and the associated space is discretized. Figure 1 shows the mesh on one of the faces of the "spherical slice" discretized. The mesh is uniform inside the cubic region at the center and within the sphere (shaded region). The rays defining the mesh in the circumferential direction (Fig. 1) are equally spaced at 11.25 degrees. Some 3-D plots are also provided in Fig. 2 to aid the visualization of the mesh. In all these figures, the mesh is not drawn to scale.

The node location on any of the axes are as follows:

A	0.0 m	B	0.0065 m	C	0.013 m	D	0.0195 m	E	0.026 m
F	0.05 m	G	0.0525 m	H	0.055 m	I	0.065 m	J	0.085 m
K	0.12 m	L	0.16 m	M	0.22 m	N	0.30 m		

These dimensions by themselves should be sufficient in defining the mesh. The dimensions above define a sphere with the inner radius of 0.05 m and an outer radius of 0.055 m. The boundaries are located about 3 sphere diameters away. The final mesh consists of 496 elements and 674 nodes.

2b. User Defined Mesh. The user is free to place the nodes as desired but should try to maintain the same total number of nodes or degrees of freedom. Adaptive mesh generators may be used here.

2c. Other Techniques. Solutions obtained using integral techniques should use matrices that have roughly the same number of non-zero elements as those meshes defined above or that have a similar mesh over the conducting region.

2d. Material Properties. The sphere is made of a material with conductivity 5.0×10^8 S/m and relative permeability 1.0. The frequency is 50 Hz.

3. Boundary Conditions

Boundary conditions should be applied on the spherical surface of the mesh to produce a uniform field of magnitude 1 T in the z direction.

4. Presentation of Results

The following quantities should be calculated and presented.

a. Total eddy current magnitude (ampere) and phase (degrees), crossing the half plane y = 0, x > 0.

b.

c. Magnetic field B_X, B_Y and B_Z: magnitude (tesla), phase (degrees) for the following points

x (m)	y (m)	z (m)
0	0	0
0.01	0	0
0.01	0.01	0.01
0.03	0.025	0.02
0.03031	0.03031	0.03031
0.030	0.031	0.032
0.065	0	0
0.1	0	0
0.1	0.11	0.12

Note: These values were chosen to give field points at mesh points, on mesh lines, at the center of elements and at other positions.

c. Current density J_X, J_Y, J_Z: magnitude (ampere/m^2) and phase (degrees) for the following points:

x (m)	y (m)	z (m)
0.0525	0	0
0.03031	0.03031	0.03031
0.01345	0.0233	0.0466

d. Global quantities: calculate the maximum and minimum values over a cycle (magnitude and phase).

Power losses (watt).

Stored energy with the conducting sphere minus the stored energy with an identical but nonconducting sphere (joule)

5. References

The solution to the sphere in uniform field may be found in many books. For example:

(1) W. R. Smythe, Static and Dynamic Electricity, McGraw Hill, 1939.

(2) M. A. Plonus, Applied Electro-Magnetics, McGraw Hill, 1978.

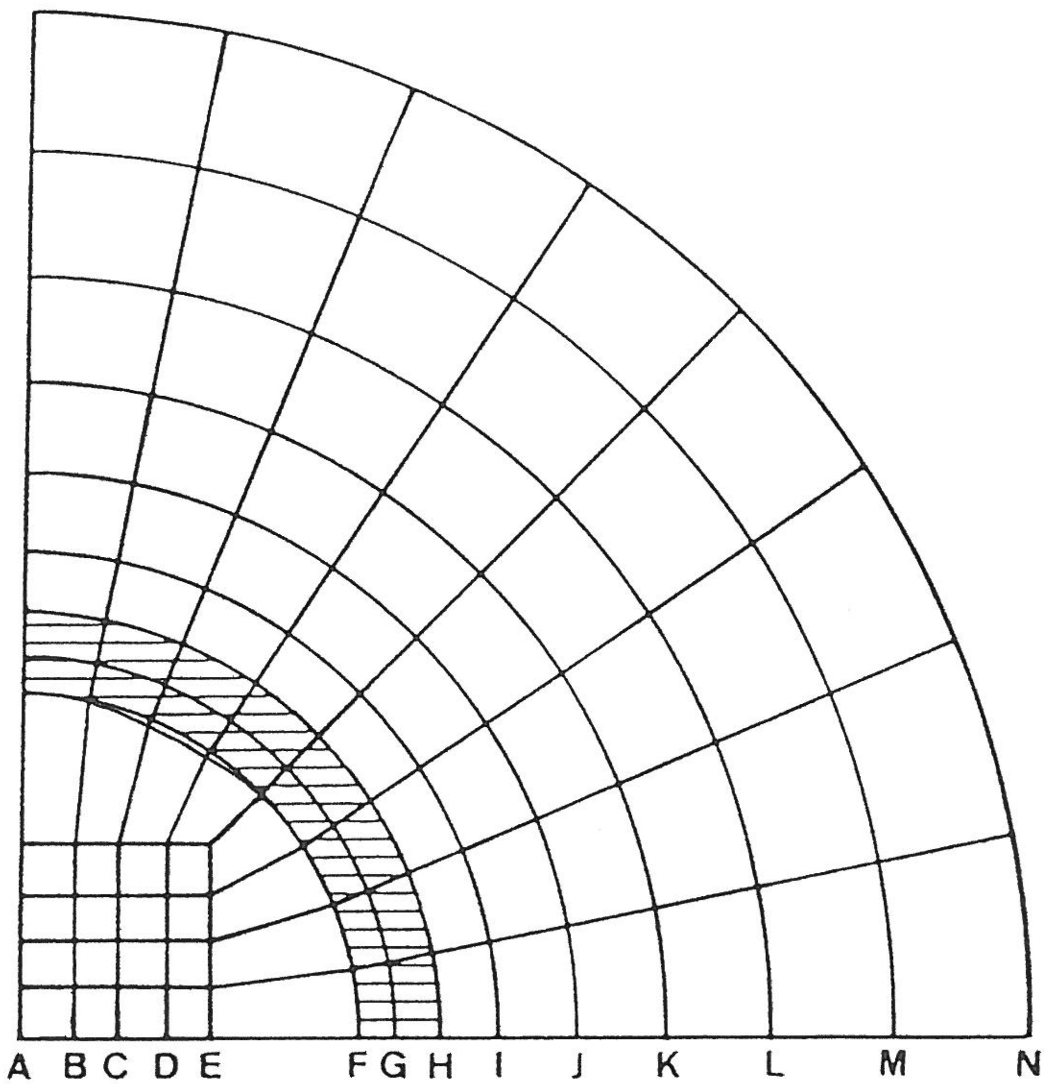

Figure 1. Coordinates of the mesh on one of the axes. The other two axes are identical. The mesh is not drawn to scale.

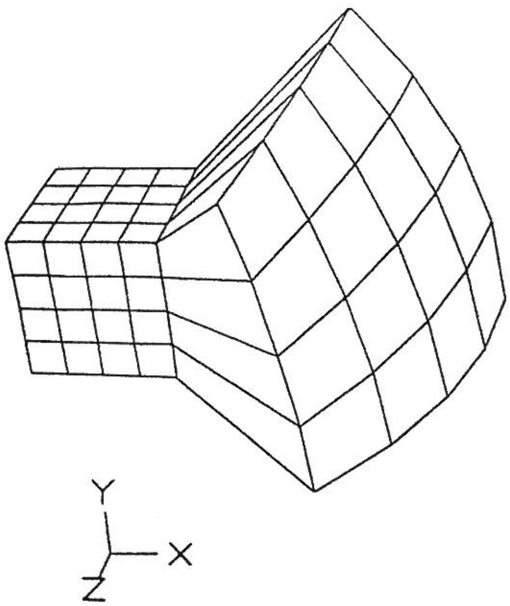

Fig. 2a. 3-D plot of the mesh in the cubic region and a portion of the mesh section connecting the cubic region and sphere.

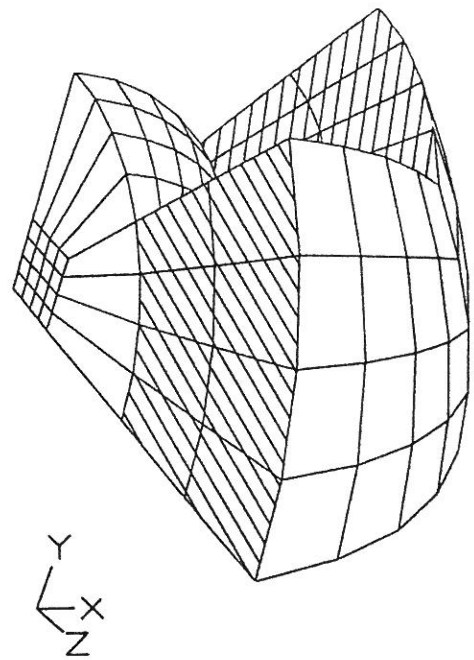

Fig. 2b. The interior region plus a portion of the conducting sphere.

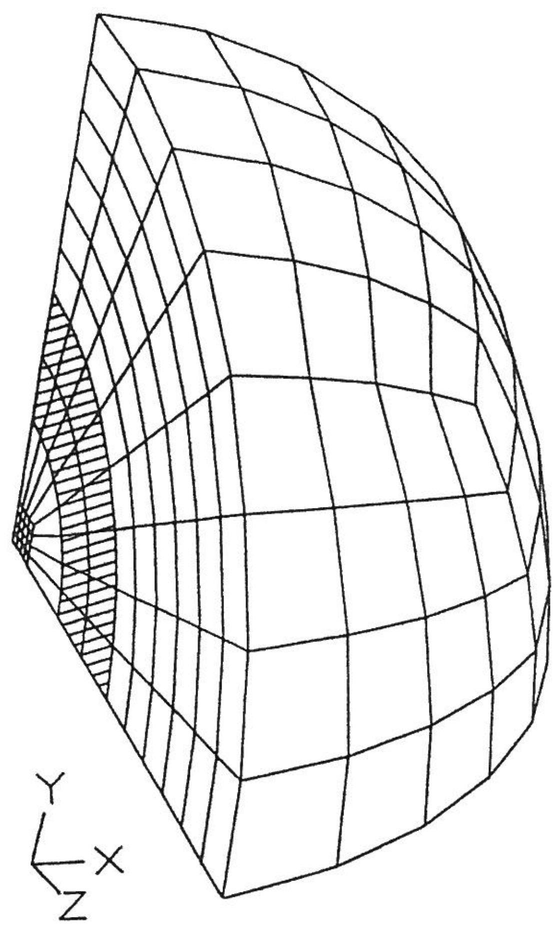

Fig. 2c. Complete mesh. Shaded region is the conducting sphere.