Problem 2
Infinitely Long Cylinder in a Sinusoidal Field

1. General Description of the Problem

This problem consists of an infinitely long hollow aluminum cylinder placed in a uniform magnetic field. The magnetic field is perpendicular to the axis of the cylinder and varies sinusoidally with time. The problem is to calculate the induced eddy currents in the aluminum and the magnetic field both inside and outside the cylinder. Global quantities such as power losses, stored energy, forces, etc. should also be calculated.

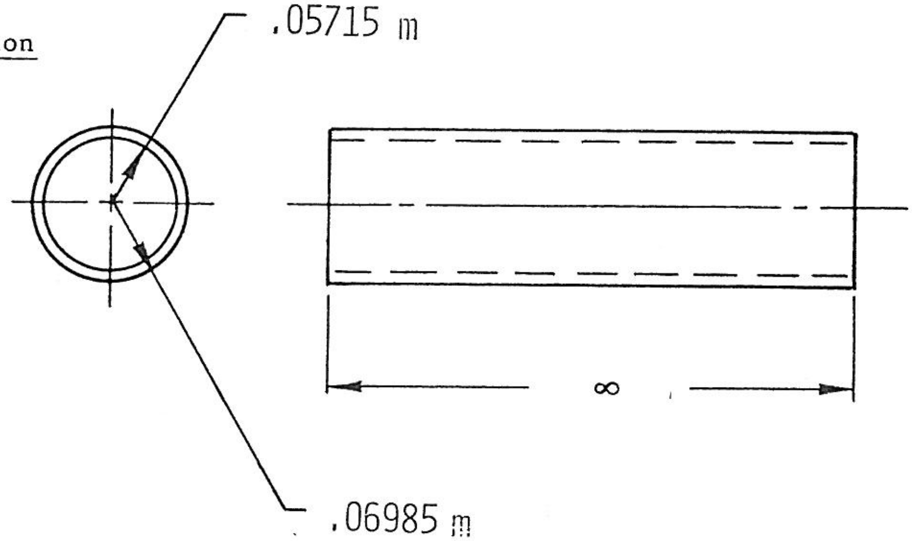
2a. Specific Mesh Description

Cylinder geometry:

Length: ∞

Inner radius: .05715 m

Outer radius: .06985 m



The cylinder material is aluminum alloy 6061 of resistivity $\rho = 3.94 \times 10^{-8}$ ohm-m. The nodes that form the specified mesh are shown in Fig. 1.a,b for the x-y plane. The finite elements need not be triangles as shown. In Fig. 1.b, the mesh has nodes at .01 m spacing in the region $0 \leq x \leq .03$, $0 \leq y \leq .03$ m. In the regions surrounding this square, the nodes on the inner radius of .05715 m are spaced 15 degrees apart. The nodes on the other three boundaries are spaced equidistant apart. Straight lines are drawn connecting the nodes on y = .03 (or x = .03) to the nodes on r = .05715. These lines have nodes equidistantly spaced. In the cylindrical region, nodes are placed at r = .05715, .05969, .06223, .06477, .06731, .06985, and .080 m at θ = 0, 15, 30, 45, 60, 75, and 90 degrees. The outer regions have nodes spaced equidistant on the boundaries. Straight lines connect the nodes at r = .08 and y = .42 (or x = .42). These lines have nodes equidistant apart. The total number of nodes is 131 and there are 105 quadrilateral elements in this x-y plane. This mesh is extended to infinity in the z direction.

If the user's mesh generator prevents generating this mesh exactly, a mesh as close to it as possible should be used.

2b. User Defined Mesh

The user is free to place the nodes as desired but should try to maintain the same total number of nodes. Adaptive mesh generators may be used here.

2c. Other Techniques

Solutions obtained using integral techniques should use matrices that have roughly the same number of non-zero elements as those meshes defined above or that have a similar mesh over the conducting region.

3. The Applied Magnetic Field and Boundary Conditions

The applied magnetic field in the y direction is uniform in space and varies sinusoidally with time as

$$B_y = B_o \cos 2\pi ft$$

where $B_o = 0.1$ T and f = 60 Hz. This condition can be imposed by choosing the proper initial boundary conditions.

Vector potentials may be specified on the r = .42 m boundary to give the desired field. If 0.0 vector potential is specified on the x = 0 plane, and (0.1) (x_i) T-m is specified at each of the 6 points on r = .42 m, where x_i is the coordinate of the point i, the field will be a uniform .1 T with no conducting cylinder. The boundary plane y = 0 is a flux normal boundary.

Scalar potentials may be specified on the y = 0 plane as 0.0 and at each of the 6 points on r = .42 m as (0.1) (y_i) T-m, where y_i is the y coordinate of the point i. The boundary plane x = 0 is a no flux crossing plane.

4. Presentation of Results

The following quantities should be calculated and presented.

a) Total eddy current in the aluminum, magnitude (ampere) phase (degrees)

b) Magnetic field B_x, B_y, magnitude (tesla), phase (degrees) for all combinations of r = 0, .01, .02, .03, .04, .05, .05842, .06858, .075, .10, .15, .20, .25, .30, .35, .40 m and θ = 0, 7.5, 14, 20, 45 degrees.

Note: These values of r and θ were chosen to give field points at mesh points, on mesh lines, at the center of elmeents and at other positions.

Global quantities: calculate the magnitude and phase.

c) Power losses in the cylinder, $\int J^2 \rho \, dS$ (watts/m)

d) Stored energy in the cylinder walls, $\int \frac{A \cdot J}{2} dS$ (joule/m)

e) Stored energy in the volume $r \leq .05715$, $\int \frac{B\ H}{2} ds$ (joule/m)

f) Stored energy in the volume $r \geq .06985$, $\int \frac{B\ H}{2} dS$ (joule/m)

g) Force on one fourth of the cylinder

$$F_x = \int J_z B_y \, dS \text{ (newton/m)}$$

$$F_y = \int J_z B_x \, dS$$

$$F_z = \int (J_x B_y - J_y B_x) \, dS$$

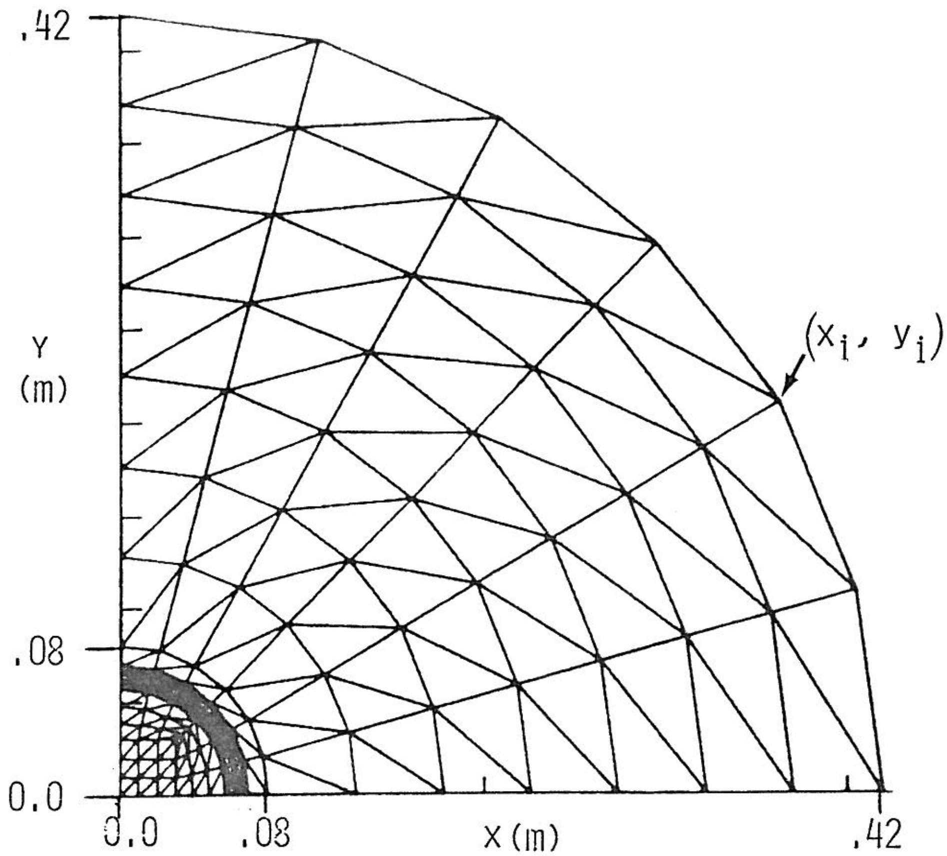


Figure 1a.

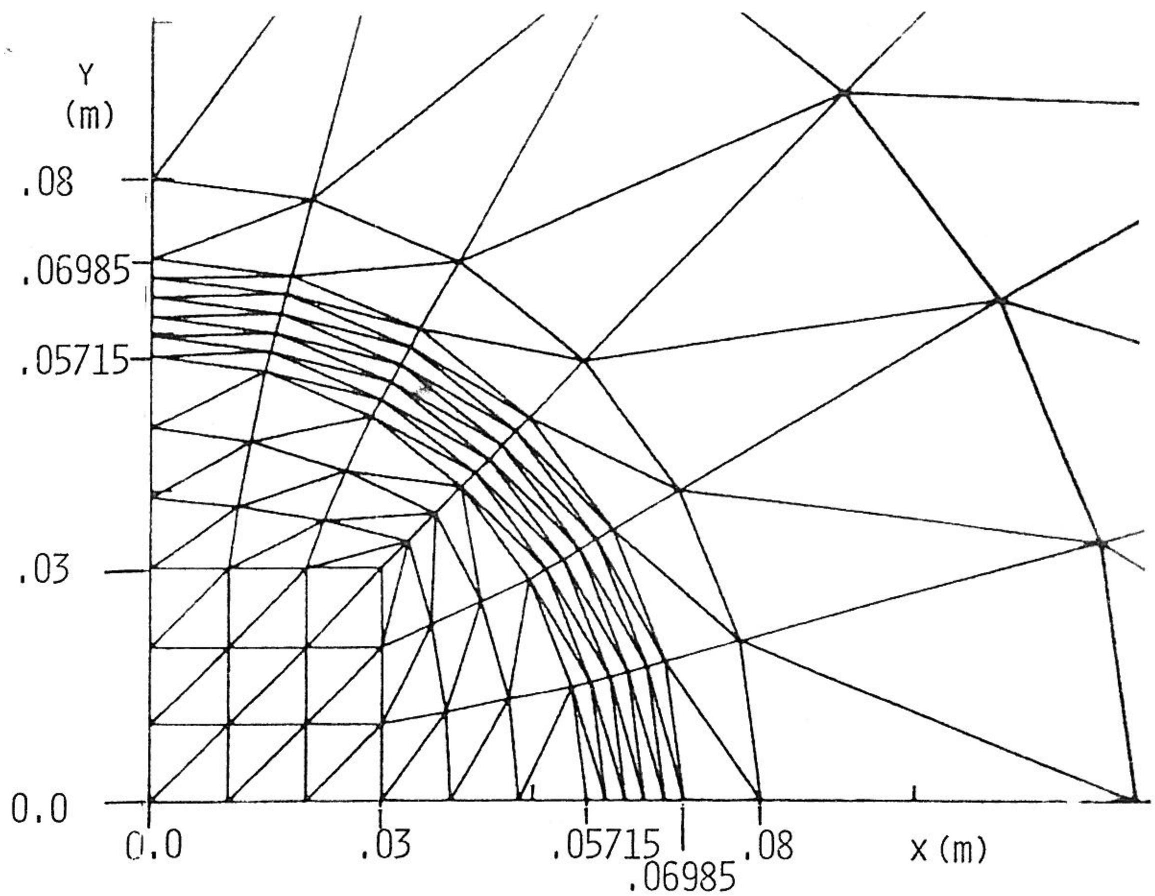


Figure 1b.