Basic equations associated to the calculation of the deformation of bodies under external efforts for linear and elastic material.

Olivier Barré, Pascal Brochet, *Member*, *IEEE*, Michel Hecquet.

***Abstract*— This paper gives an overview of the method used to calculate body deformations under external stresses. Only the 2D behaviour is taken into account. These data allow anyone to integrate this basic formulation into its finite element package as soon as the source code is available.**

***Index Terms*—External forces, Young modulus, Poisson coefficient, and Finite element method.**

I. Introduction

Any material presents deformations in front of external stresses. Such deformations can be calculated as soon as some mechanical properties are known. Hook law and especially its general form is used to perform such calculation. Using the energy principle, it is possible to link any external forces that are acting on a body to these deformations. The work of these external forces is balanced with the elastic energy variation. Using the finite element method, the problem summarizes itself in a matrix inversion. This matrix is built as an association of elementary matrix coming from the basic element. In this paper, the Hook law and the associated matrix are recalled. The definition of the elementary matrix associated to the basic element is discussed and a short example of a resolution is given.

II. Equation associated to Hook law.

Before any calculation, behaviour of the system must be examined because two different formulations exist. Generally, the usual hypothesis is the plane stresses assumption. Looking at the Fig 1a, the body is free. In Fig 1b, the body is firmly maintained along the Z direction and no deformation is allowed in such direction. This last example is known as plane deformation assumption.

O. Barré. is with the Laboratoire D'Electrotechnique et d'Electronique de Puissance of Ecole Centrale de Lille, Cité Scientifique, BP-48, 59651 Villeneuve d'Ascq Cedex, France (Phone : 33-03-20335385 ; Fax : 33-03-20335454 ; e-mail: olivier.barre@ec-lille.fr).

P. Brochet. is with the Laboratoire D'Electrotechnique et d'Electronique de Puissance of Ecole Centrale de Lille, Cité Scientifique, BP-48, 59651 Villeneuve d'Ascq Cedex, France (e-mail: pascal.brochet@ec-lille.fr).

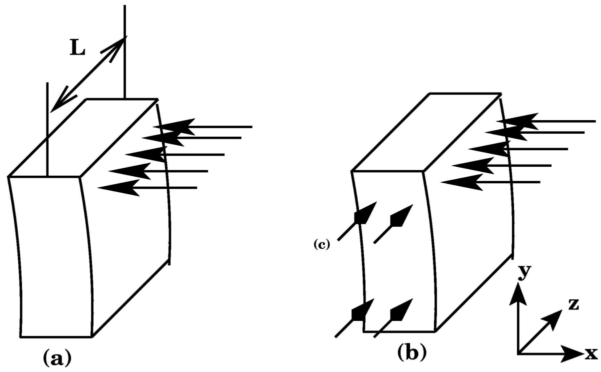

Fig. 1. Two behaviors must be considered when a body is submitted to external forces. In the Figure (a) the borders are free and a length variation along the z direction is allowed. In Figure (b), border is maintained with mechanical lock (c), hence, the preceding length variation is not allowed but stress appears in the body along the z direction.

With a structure having a z independent behaviour, three local deformations are taken into account. Submitted to external forces, a structure evolves and deformations appear. For example, in fig 2 and 3, two basic deformations are observed. Such deformations are easily understandable as directions of deformation and directions of external forces are the same. In Fig. 3, a more complex deformation appears, it can be assumed as a rotation along the Z direction. Z is orthogonal to X,Y.

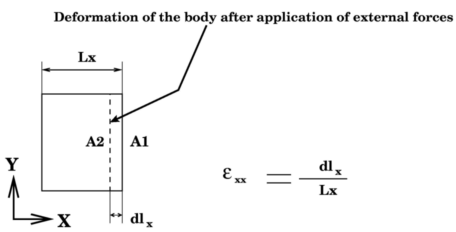

Fig. 2. Under external stresses a basic deformation along the X direction is observed.

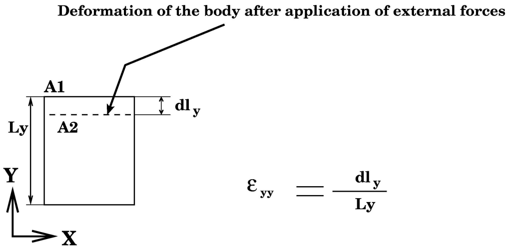

Fig. 2. Under external stresses a basic deformation along the Y direction is observed.

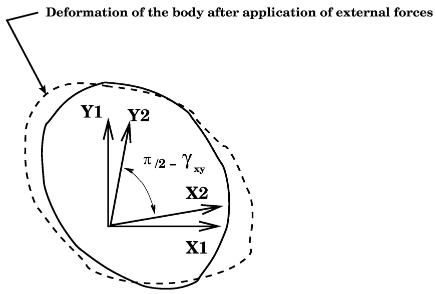

Fig. 3. Under external stresses a deformation along the z direction can be noticed. For example, the reference X1,Y1 becomes X2,Y2 after external stresses application.

With the hypothesis of elastic behaviour, the deformations are linearly linked to external stresses. In plane stresses assumption, two mechanical properties of the material are used. The modulus of elasticity (E) which is linked to the aptitude of the material to present deformations as soon as external efforts are applied and the Poisson coefficient (υ) which takes into account the ability of material to decrease its volume in front of external forces. For example, a material with a low elasticity modulus presents a high level of deformation in front of low level of external forces and for Poisson coefficient; it evolves from 0 to 0.5 (0.5 is for material where there is no volume variation under external stresses).

In 2D formulation, the three preceding deformations are taken into account. Using an X,Y,Z reference, the three stresses components are linked to these three deformations. Under plane stresses assumption, they can be joined together as it is subjected in the following equation (1) and (2).

$$\vec{\sigma} = [H]\vec{\varepsilon} \qquad (1)$$

And

$$H = \left(\frac{E}{\left(1-\upsilon^2\right)}\right)\begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & \left(\frac{1-\upsilon}{2}\right) \end{bmatrix} \qquad (2)$$

Where $\vec{\sigma}$ is the external stress, and $\vec{\varepsilon}$ the deformation of the element (3).

$$\vec{\sigma} = \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} \qquad \vec{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} \qquad (3)$$

III. Relationship beetween deformations and nodes

Equation (1) remains the main equation in finite element resolution. In this point of view, the main structure is described as an assembly of basic elements. The variations of the positions of nodes will be used to write the deformation of the basic element (Fig. 4). The equation (4) is used to link displacements associated to nodes ($\vec{u}$) to the deformation of the basic element ($\vec{\varepsilon}$).

$$\vec{\varepsilon} = [B]\vec{u} \qquad (4)$$

Where

$$[B] = \left(\frac{1}{2A}\right)\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix} \qquad (5)$$

$$2\,A = y_{31}x_{21} - x_{31}y_{21}$$

Notice that A is also the surface of the element.

$$\vec{u} = \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \end{bmatrix} \qquad (6)$$

In this formulation, $\vec{u}_{xn}$ or $\vec{u}_{yn}$ are the displacements in x or y direction of the node n. x_{nm} or y_{nm} are used in place of $x_n - x_m$ or $y_n - y_m$ and n or m are the number of the node (1,2 or 3 in the example).

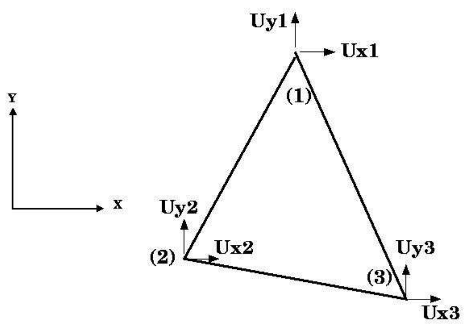

Fig. 4. Three corners define the basic element, six displacements are observed, two for each corner.

IV. RELATIONSHIP BEETWEEN EXTERNAL FORCES AND NODE DISPLACEMENTS

With the principle of energy, the work of external forces is balanced to the elastic energy variation. Equation (7) is the general form of elastic energy and $\vec{\varepsilon}^*$ is a virtual deformation. With the Hook law (1), equation (8) can be written. Using the conclusion associated to the chapter III, it is possible to describe such energy principle with a point of view associated to the basic element (9). In this equation, $\vec{u}^*$ is a virtual displacement and only the elastic energy associated to the basic element is calculated.

$$W=\int_V \vec{\varepsilon}^*.\vec{\sigma}.dV \qquad (7)$$

$$W=\int_V \vec{\varepsilon}^*.[H]\vec{\varepsilon}.dV \qquad (8)$$

$$W=\int_V \vec{u}^*.[B]^T.[H].[B]\vec{u}.dV \qquad (9)$$

With the equation (10) the work of the external forces acting on the element is defined. The equilibrium between the elastic energy and the force work implies the equations (11) and (12).

$$W=\int_S \vec{u}^*.\vec{F}.ds \qquad (10)$$

$$\int_V \vec{u}^*[B]^T.[H].[B]\vec{u}.dv=\int_S \vec{u}^*.\vec{F}.ds \qquad (11)$$

$$\int_V [B]^T.[H].[B]\vec{u}.dv=\int_S \vec{F}.ds \qquad (12)$$

It appears that it is now possible to link external forces acting on one element to the displacement of the nodes (12) (Fig. 5). In this equation S_e is the volume of the element in 3D modelling or the surface the element ($S_e=A$) in 2D modelling.

$$[B]^T.[H].[B].S_e \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \end{bmatrix} = \begin{bmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \\ f_{x3} \\ f_{y3} \end{bmatrix} \qquad (12)$$

$$\begin{bmatrix} m_{11} & .. & .. & .. & .. & m_{16} \\ .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. \\ m_{61} & .. & .. & .. & .. & m_{66} \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \end{bmatrix} = \begin{bmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \\ f_{x3} \\ f_{y3} \end{bmatrix} \qquad (13)$$

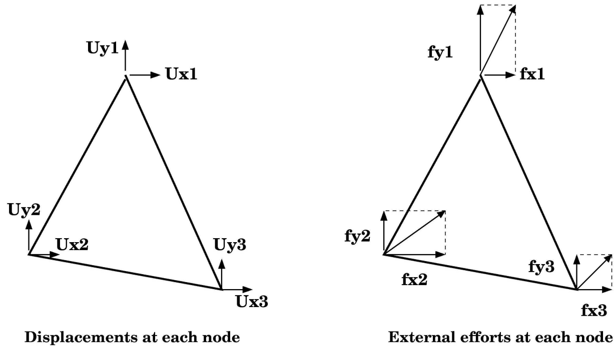

Fig. 5. Notations used for external efforts and node displacements.

Hence, as soon as the modulus elasticity and the Poisson coefficient are known, the H matrix can be calculated. For each basic element that composes the structure, the B matrix can be also calculated. In conclusion, the $[B]^T[H][B].Se$ matrix is defined for each basic element.

V. FINAL ASSEMBLY AND RESOLUTION.

The structure is described in term of nodes and external forces applied on these nodes. At each node, two displacements and two external efforts are attached. The following notation will be used in the example describing the whole structure: at node I or J, U_{ix} and U_{iy}, or U_{jx} and U_{jy}, are the associated displacements. F_{ix} and F_{iy}, or F_{jx} and F_{jy}, are the associated external efforts. Relation between external efforts and displacements can be expressed with the equation (13).

.

$$[K].\begin{bmatrix} U_{0x} \\ U_{0y} \\ .. \\ .. \\ U_{ix} \\ U_{iy} \\ .. \\ .. \\ U_{jx} \\ U_{jy} \\ .. \\ .. \\ U_{nx} \\ U_{ny} \end{bmatrix} = \begin{bmatrix} F_{0x} \\ F_{0y} \\ .. \\ .. \\ F_{ix} \\ F_{iy} \\ .. \\ .. \\ F_{jx} \\ F_{jy} \\ .. \\ .. \\ F_{nx} \\ F_{ny} \end{bmatrix} \qquad (13)$$

This general matrix is obtained with a concatenation of elementary matrixes coming from each basic element. In addition, it appears that some values can be imposed. At any node where there is no external effort, F_{ix} and F_{iy} are equal to 0 and at any node where there is contact, U_{ix} and U_{iy} are equal to 0. For example, using Fig. 6, equation (13),(14),(15) (16) and (17) can be added.

$$F_{jx}=F0_x \quad F_{jy}=F0_y \tag{13}$$

$$F_{ix}=0 \quad F_{iy}=0 \tag{14}$$

but

$$U_{ix}\neq 0 \quad U_{iy}\neq 0 \tag{15}$$

$$U_{kx}=0 \quad U_{ky}=0 \tag{16}$$

but

$$F_{kx}\neq 0 \quad F_{ky}\neq 0 \tag{17}$$

When the external efforts are given, it is possible to calculate each displacement at each node in doing a matrix inversion of K.

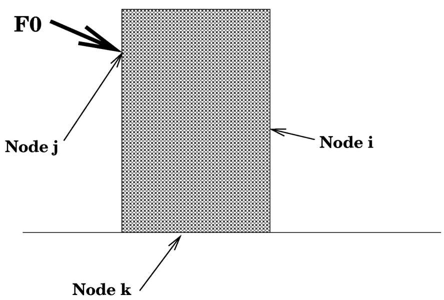

Fig. 6. Example of a structure in real situation. For nodes i, j, and k external efforts or displacement can be imposed.

VI. Example:

In order to conclude this short description of the mechanical calculation, an example is treated (Fig. 7).

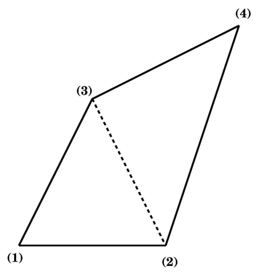

Fig. 7. Magnetic core built from parts (A) and (B) .

This small structure is defined by four nodes (Tab.1). As an example, this figure is modelled with only two elements. Three nodes define each element and for each element the position of each node is given (Tab.2).

Tab.1

Node	X (m)	Y (m)
1	0	0
2	0.1	0
3	0.05	0.1
4	0.15	0.15

Tab. 2

Element 1		Element 2	
Node 1	X1= 0 m	Node 1	X1=0.1 m
	Y1= 0 m		Y2=0.1 m
Node 2	X2=0.1 m	Node 2	X2=0.05 m
	Y2=0 m		Y2=0.1 m
Node 3	X3=0.05 m	Node 3	X3=0.15 m
	Y3=0.1m		Y3=0.15 m

1) *Step 1 (Element characteristics)*

For each element the matrixes B and B^T are calculated (18),(19),20,

a)Element 1 (Fig. 8).

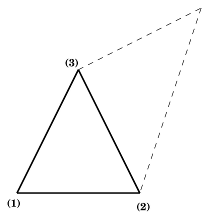

Fig. 8. First element associated to the main structure.

$$[B_1]=\left(\frac{1}{2A_1}\right)\begin{bmatrix} -0.1 & 0 & 0.1 & 0 & 0 & 0 \\ 0 & -0.05 & 0 & -0.05 & 0 & 0.1 \\ -0.05 & -0.1 & -0.05 & 0.1 & 0.1 & 0 \end{bmatrix} \tag{18}$$

$$[B_1^T]=\left(\frac{1}{2A_1}\right)\begin{bmatrix} -0.1 & 0 & -0.05 \\ 0 & -0.05 & -0.1 \\ 0.1 & 0 & -0.05 \\ 0 & -0.05 & 0.1 \\ 0 & 0 & 0.1 \\ 0 & 0.1 & 0 \end{bmatrix} \tag{19}$$

And

$$2A_1=(0.1)(0.1)-(0.05)(0)=0.01$$
$$A_1=0.005$$

b) Element 2 (Fig. 9).

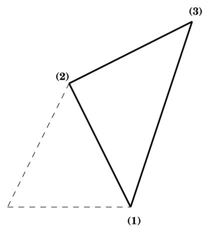

Fig. 9. Second element associated to the main structure.

$$[B_2]=\left(\frac{1}{2A_2}\right)\begin{bmatrix} -0.05 & 0 & 0.15 & 0 & -0.1 & 0 \\ 0 & 0.1 & 0 & -0.05 & 0 & -0.05 \\ 0.1 & -0.05 & -0.05 & 0.15 & -0.05 & -0.1 \end{bmatrix} \quad (20)$$

$$[B_2]=\left(\frac{1}{2A_2}\right)\begin{bmatrix} -0.05 & 0 & 0.1 \\ 0 & 0.1 & -0.05 \\ 0.15 & 0 & -0.05 \\ 0 & -0.05 & 0.15 \\ -0.1 & 0 & -0.05 \\ 0 & -0.05 & -0.1 \end{bmatrix} \quad (21)$$

And

$$2A_2=(-0.05)(0.15)-(0.05)(0.1)=-0.0125$$
$$A_2=0.00625$$

2) Step 2 (Material)

With properties of the material, the elasticity matrix, H, can be built. For this example, the elasticity modulus is E=100 000 MPA (1 MPA = 10^6 PA and 1 PA = 1 N / m²) and the Poisson coefficient $\upsilon=0.3$. Under plane stresses assumption, this matrix is (22).

$$H=\left(\frac{100000.10^6}{\left(1-0.3^2\right)}\right)\begin{bmatrix} 1 & 0.3 & 0 \\ 0.3 & 1 & 0 \\ 0 & 0 & \left(\frac{1-0.3}{2}\right) \end{bmatrix} \quad (22)$$

$$H=\begin{bmatrix} 109\,890.10^6 & 32967.10^6 & 0 \\ 32967.10^6 & 109890.10^6 & 0 \\ 0 & 0 & 38461.10^6 \end{bmatrix} \quad (23)$$

This matrix is the same for all elements.

3) Step 3 (Matrix associated to each element)

For each element, the $[B^T]$ [H] [B]. S_e entity can be expressed. It is mane K1 for the first element and K2 for the second element (24), (25).

$$K1=[B_1^T]\,[H]\,[B_1]\,.\,A_1 \quad (24)$$

K1= 1.0e+010 *

5.9753	1.7857	-5.0137	-0.1374	-0.9615	-1.6484
1.7857	3.2967	0.1374	-0.5494	-1.9230	-2.7473
-5.0137	0.1374	5.9753	-1.7857	-0.9615	1.6484
-0.1374	-0.5494	-1.7857	3.2967	1.9230	-2.7473
-0.9615	-1.9230	-0.9615	1.9230	1.9230	0
-1.6484	-2.7473	1.6484	-2.7473	0	5.4945

$$K2=[B_2^T]\,[H]\,[B_2]\,.\,A_2 \quad (25)$$

K2 = 1.0e+011 *

0.2637	-0.1429	-0.4066	0.2637	0.1429	-0.1209
-0.1429	0.4780	0.2363	-0.3352	-0.0934	-0.1429
-0.4066	0.2363	1.0275	-0.2143	-0.6209	-0.0220
0.2637	-0.3352	-0.2143	0.4560	-0.0494	-0.1209
0.1429	-0.0934	-0.6209	-0.0494	0.4780	0.1429
-0.1209	-0.1429	-0.0220	-0.1209	0.1429	0.2637

4) Step 4 (Building the entire matrix)

$$\begin{bmatrix} K_{11} & .. & .. & .. & .. & .. & .. & K_{18} \\ .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. \\ K_{81} & .. & .. & .. & .. & .. & .. & K_{88} \end{bmatrix}\begin{bmatrix} U_{1x} \\ U_{1y} \\ U_{2x} \\ U_{1y} \\ U_{3x} \\ U_{3y} \\ U_{4x} \\ U_{4y} \end{bmatrix}=\begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \\ F_{4x} \\ F_{4y} \end{bmatrix} \quad (26)$$

This matrix (26) is the entire matrix associated to the whole structure. It must be noticed that displacement U_{xy} of the whole structure can be associated to nodes describing the elements (Tab. 3). With this information, it immediately appears that the node 3 of the entire structure is also the node 3 for the element 1 and the node 2 for the element 2. The displacement associated to this node is the same for each element.

Tab. 3

Node (entire structure)		Element 1		Element 2	
Node 1	U_{1x}	Node 1	u_{1x}		
	U_{1y}		u_{1y}		
Node 2	U_{2x}	Node 2	u_{2x}	Node 1	u_{1x}
	U_{2y}		u_{2y}		u_{1y}
Node 3	U_{3x}	Node 3	u_{3x}	Node 2	u_{2x}
	U_{3y}		u_{3y}		u_{2y}
Node 4	U_{4x}			Node 3	u_{3x}
	U_{4y}				u_{3yx}

Using the notation associated to the whole structure, the relation between the displacements and the external forces can be rewritten.

$$K1.\begin{bmatrix} U_{1x} \\ U_{1y} \\ U_{2x} \\ U_{2y} \\ U_{3x} \\ U_{3y} \end{bmatrix} = \begin{bmatrix} f_{1x} \\ f_{1y} \\ f_{2x} \\ f_{2y} \\ f_{3x} \\ f_{4y} \end{bmatrix} \qquad K2.\begin{bmatrix} U_{2x} \\ U_{2y} \\ U_{3x} \\ U_{3y} \\ U_{4x} \\ U_{4y} \end{bmatrix} = \begin{bmatrix} f_{2x} \\ f_{2y} \\ f_{3x} \\ f_{3y} \\ f_{4x} \\ f_{4y} \end{bmatrix} \tag{27}$$

The matrix K can be obtained as an association of K1 and K2 (27), (28).

$$\begin{bmatrix} K1_{11} & K1_{12} & K1_{13} & K1_{14} & K1_{15} & K1_{16} & 0 & 0 \\ K1_{21} & K1_{22} & K1_{23} & K1_{24} & K1_{25} & K1_{26} & 0 & 0 \\ K1_{31} & K1_{32} & K1_{33}+K2_{11} & K1_{34}+K2_{12} & K1_{35}+K2_{13} & K1_{36}+K2_{14} & K2_{15} & K2_{16} \\ K1_{41} & K1_{42} & K1_{43}+K2_{21} & K1_{44}+K2_{22} & K1_{45}+K2_{23} & K1_{46}+K2_{24} & K2_{25} & K2_{26} \\ K1_{51} & K1_{52} & K1_{53}+K2_{31} & K1_{54}+K2_{32} & K1_{55}+K2_{33} & K1_{56}+K2_{34} & K2_{35} & K2_{36} \\ K1_{61} & K1_{62} & K1_{63}+K2_{41} & K1_{64}+K2_{42} & K1_{65}+K2_{43} & K1_{66}+K2_{44} & K2_{45} & K2_{46} \\ 0 & 0 & K2_{51} & K2_{52} & K2_{53} & K2_{54} & K2_{55} & K2_{56} \\ 0 & 0 & K2_{61} & K2_{62} & K2_{63} & K2_{64} & K2_{65} & K2_{66} \end{bmatrix}$$

(28)

Forces associated to nodes connected to each other disappear. Only remain external forces associated to the structure (29). In this equation F_{xy} are external forces.

$$\begin{bmatrix} K_{11} & .. & .. & .. & .. & .. & .. & K_{18} \\ .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. \\ K_{81} & .. & .. & .. & .. & .. & .. & K_{88} \end{bmatrix} \begin{bmatrix} U_{1x} \\ U_{1y} \\ U_{2x} \\ U_{1y} \\ U_{3x} \\ U_{3y} \\ U_{4x} \\ U_{4y} \end{bmatrix} = \begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ 0 \\ 0 \\ F_{4x} \\ F_{4y} \end{bmatrix} \tag{29}$$

The structure is fixed on the floor using the node 1 and the node 2; consequently, the displacements associated to these nodes are equal to 0 and the preceding equation (29) can be modified (30).

$$\begin{bmatrix} K_{11} & .. & .. & .. & .. & .. & .. & K_{18} \\ .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. \\ K_{81} & .. & .. & .. & .. & .. & .. & K_{88} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ U_{3x} \\ U_{3y} \\ U_{4x} \\ U_{4y} \end{bmatrix} = \begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ 0 \\ 0 \\ F_{4x} \\ F_{4y} \end{bmatrix} \tag{30}$$

When a force is applied to node 4, F_{4x} and F_{4y} are known and the entire system contains 8 unknown elements and 8 equations. Hence this system can be solved. F_{1x}, F_{1y}, F_{2x}, F_{2y}, the action of the floor on the node 1 and 2 can be calculated and U_{3x} , U_{3y}, U_{4x}, U_{4y}, the displacements of the two free nodes can also be calculated at the same time. The problem can be written in a more useful form (31).

$$[K']\begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ U_{3x} \\ U_{3y} \\ U_{4x} \\ U_{4y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ F_{4x} \\ F_{4y} \end{bmatrix} \; : \; [K'][R] = [S] \tag{31}$$

and K' is :

$$\begin{bmatrix} -1 & 0 & 0 & 0 & K1_{15} & K1_{16} & 0 & 0 \\ 0 & -1 & 0 & 0 & K1_{25} & K1_{26} & 0 & 0 \\ 0 & 0 & -1 & 0 & K1_{35}+K2_{13} & K1_{36}+K2_{14} & K2_{15} & K2_{16} \\ 0 & 0 & 0 & -1 & K1_{45}+K2_{23} & K1_{46}+K2_{24} & K2_{25} & K2_{26} \\ 0 & 0 & 0 & 0 & K1_{55}+K2_{33} & K1_{56}+K2_{34} & K2_{35} & K2_{36} \\ 0 & 0 & 0 & 0 & K1_{65}+K2_{43} & K1_{66}+K2_{44} & K2_{45} & K2_{46} \\ 0 & 0 & 0 & 0 & K2_{53} & K2_{54} & K2_{55} & K2_{56} \\ 0 & 0 & 0 & 0 & K2_{63} & K2_{64} & K2_{65} & K2_{66} \end{bmatrix}$$

$K1_{xy}$ and $K2_{xy}$ are calculated in the step 3. As soon as the external efforts are given, S is totally known and R is the unknown entity. R is immediately calculated with a matrix inversion, R = $[K']^{-1}$ S. For example, $F_{4x} = 10^{6}$ N/m and $F_{4y} = 10^{6}$ N/m, the results for the displacement to the node 3 and 4 are given in Tab.4.

Tab. 4

U_{3x}	$3.4891\ 10^{-5}$ m (0.03489 mm)
U_{3y}	$1.1977\ 10^{-5}$ m (0.01197 mm)
U_{4x}	$6.4\ 10^{-5}$ m (0.064mm)
U_{4y}	$1.1642\ 10^{-5}$ m (0.01164mm)

And the force at the node 1 and 2 in Tab. 5.

Tab. 5

F_{1x}	$-5.3292\ 10^{5}$
F_{1y}	$-1.0000\ 10^{5}$
F_{2x}	$-4.6707\ 10^{5}$
F_{2y}	$0.0007\ 10^{5}$

With commercial software, and the same mesh, the results are Tab. 6 and Tab. 7.

Tab. 6

U_{3x}	0.03488 mm
U_{3y}	0.01198 mm
U_{4x}	0.06398 mm
U_{4y}	0.01165 mm

Tab. 7

F_{1x}	$-5.32918\ 10^5$
F_{1y}	$-1.0000\ 10^5$
F_{2x}	$-4.67081\ 10^5$
F_{2y}	$0.0000\ 10^5$

In conclusion, this short example shows the procedure used to calculate the deformation of a body with a 2D behaviour. Basic element with 3 nodes is used to model the structure. The external force is acting only on node 4 (Fig.10). Deformation of the body and displacement of the nodes are calculated (Fig. 11) and (Tab. 6). Action of the floor on the nodes can also be defined (Fig. 12) and (Tab. 7). Additional information about basic element can be found in reference book such as [1],[2],[3] and [4]

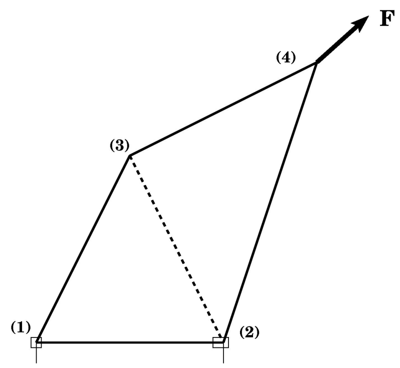

Fig. 10. External effort acting on studied structure. $Fx=10^6$ N/m and $Fy=10^6$ N/m.

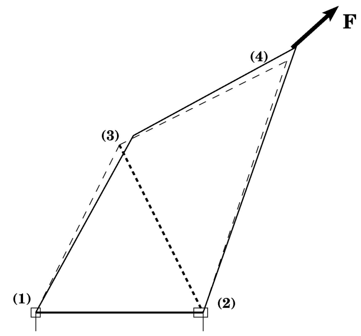

Fig. 11. Deformation of the structure.

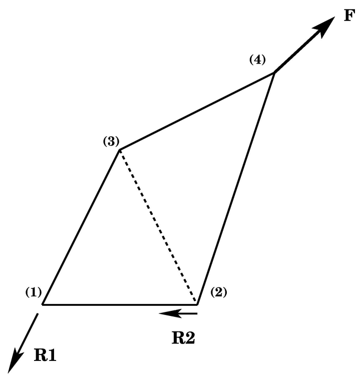

Fig. 12. Action of the floor on the structure.

REFERENCES

[1] Bathe, K.J., “Finite element procedures in engineering analysis”, Prentice Hall 1982.

[2] Kardestuncer H., Norrie D., “Finite element handbook”, McGraw-Hill, 1987.

[3] Zienkiewicz O.C., “The finite element method”, 3rd edition, McGraw-Hill 1977.

[4] Zienkiewicz O.C., Taylor R.L., “The finite element method” 4th edition, Volume 1: Basic formulations and linear problems, McGRAW-Hill, 1989.