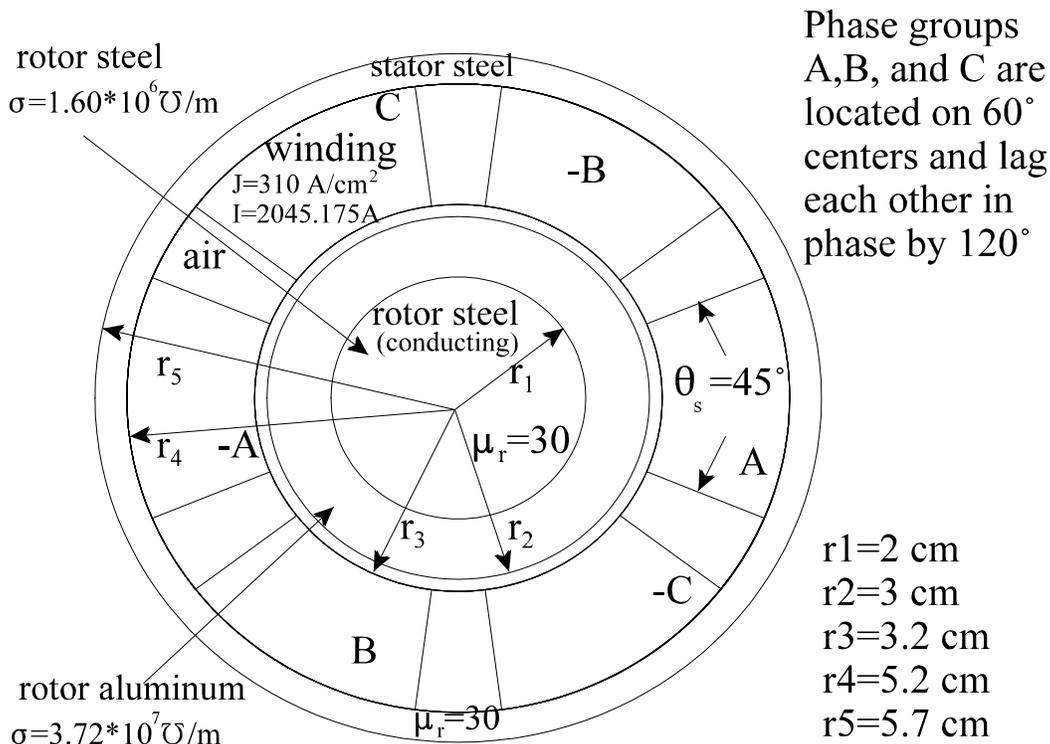


**Induction Motor Analyses**  
**InternationalTEAM Workshop Problem 30**

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Abstract - This document describes an induction motor problem in which the eddy currents in the rotor are induced both by time harmonic currents on the stator and by the rotation of the rotor. Although the problem is two-dimensional, it presents some interesting challenges heretofore not addressed in existing TEAM problems. Among those is the existence of the  $\vec{\omega} \times \vec{B}$  induced electric field and the high rotation speeds. In a volume based finite element or finite difference code, this term must be coded into the algorithm. At high rotation speeds, traditional approaches involving upwinding techniques are challenged. Boundary element codes are able to modify the Green's function to incorporate pure translation induced eddy currents; with rotation, there is yet no known technique for incorporating rotation effects into the Green's function.

Introduction



**Figure 1** Three Phase induction motor problem with a 45 degree winding spread per phase, holding  $J$  constant at 310 A/cm<sup>2</sup>.

Two induction motor problems are to be analyzed. The first, shown in Figure 1, is that of a three phase exposed winding motor. Each stator winding phase spans 45°. The current density is maintained constant at 310 A/cm<sup>2</sup>. The object is to predict the torque, power dissipated, and

stator terminal voltage induced for rotor speed speeds ranging from 0 to 1200 rad/s, roughly three times faster than the stator field speed of 377 rad/s. The three phase winding is excited at 60 Hz. Both the rotor and stator steel has a relative permeability  $\mu_r=30$ . The stator steel is laminated and has a conductivity  $\sigma=0$ ; the rotor steel has a conductivity  $\sigma=1.6*10^6 \text{ } \Omega/\text{m}$ . The rotor aluminum has a conductivity  $\sigma=3.72*10^7 \text{ } \Omega/\text{m}$ . In addition to torque, voltage, and power, the radial B field and azimuthal H field are to be determined along the x-axis between  $r_3$  and  $r_4$  ( $r_3 \leq x \leq r_4$ ).

Each of these quantities are computed analytically. The primary quantities, torque, voltage, and power dissipation are displayed in Table I. The forth column represents the total rotor loss in both the aluminum and the rotor steel. All quantities are computed on a per unit depth (1 m) basis. The final column represents just the rotor steel loss due to  $I^2R$  dissipation. The induced voltage in the phase A coil is computed as if the stator winding were comprised of a single turn.

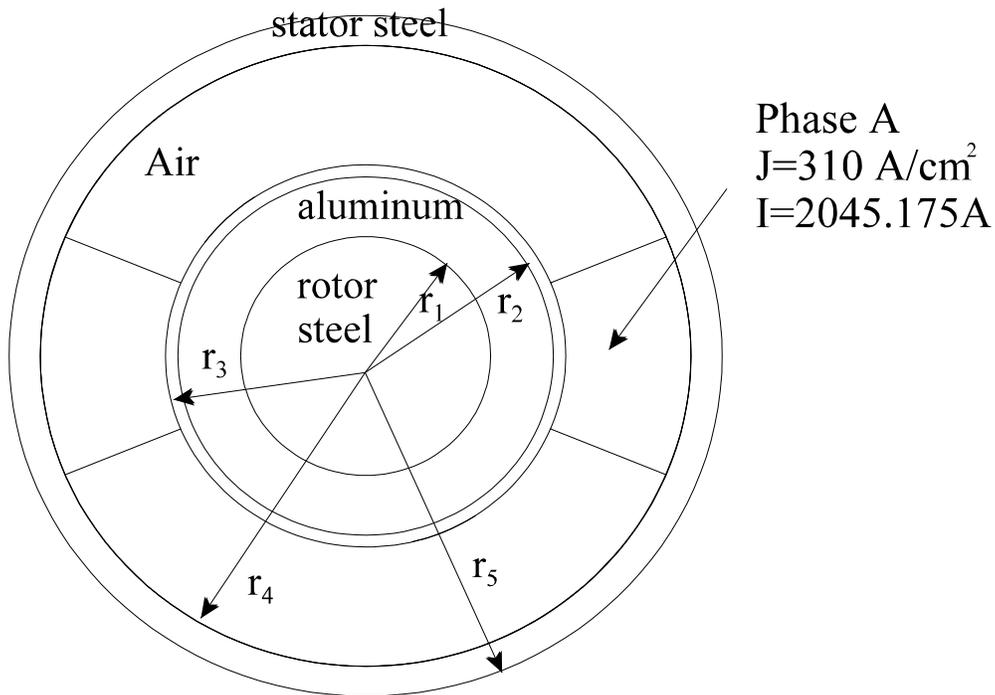
Table I Three phase predictions of torque, voltage, and power dissipation.

speed $\Omega_r$ (rad/s)	Torque (N/m)	Voltage /turn (V/m/turn)	Rotor Loss (W/m)	Steel Loss (W/m)
0	3.825857	0.637157	1455.644	17.40541
200	6.505013	0.845368	1179.541	16.98615
400	-3.89264	1.477981	120.0092	1.383889
600	-5.75939	0.76176	1314.613	17.87566
800	-3.59076	0.617891	1548.24	16.88702
1000	-2.70051	0.575699	1710.686	14.32059
1200	-2.24996	0.556196	1878.926	12.01166

The radial B field and  $\theta$  directed H field are also predicted at 200 rad/s on the x axis between  $r_3$  and  $r_4$ . Table II' shows the tabulated results.

Table II' Radial B field and azimuthal H field for the three phase motor at  $\Omega_r=200$  rad/s.

X (m)	Real( $B_r$ ) T	Imag( $B_r$ ) T	Real( $H_\theta$ ) A/m	Imag( $H_\theta$ ) A/m
0.032	0.018854	0.016392	-46504.9	-10757.6
0.034222	0.017122	0.017079	-39165.7	-8939.18
0.036444	0.015643	0.017412	-32564.6	-7462.55
0.038667	0.014375	0.017455	-26499.5	-6253.55
0.040889	0.013284	0.017266	-20817.4	-5250.13
0.043111	0.012341	0.016895	-15405.7	-4406.77
0.045333	0.011522	0.016385	-10181.6	-3690.02
0.047556	0.010807	0.015772	-5084.64	-3074.86
0.049778	0.010179	0.015085	-71.2009	-2542.22
0.052	0.009625	0.014347	4888.668	-2077.37



**Figure 2** Single phase induction motor problem excited at 60 Hz.



The second problem shown in Figure 2 is that of a single phase induction motor. The winding is excited at 60 Hz. The objective is to compute the torque-speed curve for a rotor speed ranging from 0 to 358 rad/s (0.95% of peak field speed). Researchers who have attempted to work with single phase induction motors know of the difficulties of obtaining an accurate torque prediction; this torque results qualitatively from the subtraction of the effect of two counter rotating traveling waves. Table III) shows the torque, voltage, and power dissipation for the single phase machine.

Table III) Torque, voltage , and power dissipation in the single phase motor of Figure 2.

speed $\Omega_r$ (rad/s)	Torque (N/m)	Voltage (V/m/turn)	Rotor loss (W/m)	Steel loss (W/m)
0	0	0.536071	341.7676	3.944175
39.79351	0.052766	0.537466	341.2465	3.933111
79.58701	0.096143	0.541495	340.4618	3.900878
119.3805	0.14305	0.548603	340.0396	3.848117
159.174	0.19957	0.560074	340.225	3.767681
198.9675	0.2754	0.578808	339.2994	3.635357
238.761	0.367972	0.609649	333.6163	3.404092
278.5546	0.442137	0.658967	317.9933	2.999715
318.3481	0.375496	0.728552	288.079	2.355622
358.1416	-0.0707	0.790068	256.6437	1.674353

### Conclusions

The torque, induced Phase A voltage, and rotor power dissipation are presented for a three phase and single phase motor for various speeds. The desired output quantities are those presented in Tables I, II, and III.