SMES Optimization Benchmark: TEAM Workshop Problem 22

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1 **General Description**

Two concentric coils carrying current with opposite direction (Fig. 1) and running under superconducting conditions offer the opportunity to store a significant amount of energy in their magnetic fields while keeping the stray field within certain limits [1]. An optimal design of the system should therefore couple the desired value of energy to be stored (first objective) with a minimal stray field (second objective) [2]. This problem has been accepted as benchmark problem TEAM problem 22 [3].

A discrete problem with 3 parameters (radius, height and thickness of the outer coil) was defined and



Figure 1: SMES Configuration

completely enumerated for a given scalar objective function. Additionally, an 8 parameters problem (radius, height, thickness and current density of both coils) was defined. Common to both problems the objective function was defined as a weighted sum of the two objectives (1).

Independently from the number of parameters a Superconducting Magnetic Energy Storage (SMES) configuration (Fig. 1) shall be optimized with respect to the following objectives:

- 1. The stored energy in the device should be 180 MJ.
- 2. The magnetic field must not violate a certain physical condition which guarantees superconductivity (quench condition (6)).
- 3. The stray field (measured at a distance of 10 meters from the device) should be as small as possible.

2 Definition of the 3 Parameter Problem, Discrete Case

$h_{1}/2$ J_1 R_1 R_2 $h_2/2$ J_2 d_1 d_2 A/mm^2 A/mm^2 m m m m m m 2.60.204 0.1 \min ---_ max 3.41.10.4_ -_ step size 0.01-0.007 0.003-_ _ _ # of values 81 129101 _ _ _ _ fixed 2.0_ 0.8_ 0.27_ 22.5-22.5

2.1 Design Parameters

2.2 Objective Function

The objective function of this problem has to take both the energy requirement (E should be as close as possible to 180 MJ) and the stray field requirements (B_{stray} evaluated along 22 equidistant points along line a and line b in Fig. 1 as small as possible) into account, hence the problem is a multi objective problem. However, the two objectives are mapped into a single objective function (1).

$$OF = \frac{B_{stray}^2}{B_{norm}^2} + \frac{|E - E_{ref}|}{E_{ref}},\tag{1}$$

where $E_{ref} = 180 \text{ MJ}$, $\mathbf{B_{norm}} = \mathbf{3} \ \mu \text{T}$ and B_{stray}^2 is defined as :

$$B_{stray}^{2} = \frac{\sum_{i=1}^{22} |\mathbf{B}_{stray,i}|^{2}}{22}.$$
 (2)

3 Definition of the 8 Parameter Problem, Continuous Case

3.1 Geometrical constraints

| | R_1 | R_2 | $h_{1}/2$ | $h_{2}/2$ | d_1 | d_2 | J_1 | J_2 |
|--------|-------|-------|-----------|-----------|-------|-------|----------|----------|
| | m | m | m | m | m | m | A/mm^2 | A/mm^2 |
| min | 1.0 | 1.8 | 0.1 | 0.1 | 0.1 | 0.1 | 10 | -30 |
| \max | 4.0 | 5.0 | 1.8 | 1.8 | 0.8 | 0.8 | 30 | -10 |



Figure 2: Critical curve of an industrial superconductor.

3.2 Objective Function

The objective function of this problem has to take both the energy requirement (E should be as close as possible to 180 MJ) and the stray field requirements (B_{stray} evaluated along 22 equidistant points along line a and line b in Fig. 1 as small as possible) into account, hence the problem is a multi objective problem. However, the two objectives are mapped into a single objective function (3).

$$OF = \frac{B_{stray}^2}{B_{norm}^2} + \frac{|E - E_{ref}|}{E_{ref}},\tag{3}$$

where $E_{ref} = 180 \text{ MJ}$, $\mathbf{B_{norm}} = \mathbf{200} \ \mu \text{T}$ and B_{stray}^2 is defined as :

$$B_{stray}^{2} = \frac{\sum_{i=1}^{22} |\mathbf{B}_{stray,i}|^{2}}{22}.$$
 (4)

4 Constraints

4.1 Design Constraints

The solenoids should not overlap each other (5).

$$R_1 + \frac{d_1}{2} < R_2 - \frac{d_2}{2} \tag{5}$$

4.2 Quench Condition

The superconducting material should not violate the quench condition that links together the value of the current density and the maximum value of magnetic flux density, as shown in Fig 2. The critical curve has been approximated by (6)

The critical curve has been approximated by (6).

$$|\mathbf{J}| = (-6.4|\mathbf{B}| + 54.0) \text{ A/mm}^2 \tag{6}$$

5 Results

5.1 3 Parameter Problem, Discrete Case

Best result of the 3 Parameter Problem calculated with (1) and (2) with a strayfield of $B_{norm} = 3 \mu T$:

| | R_1 | R_2 | $h_{1}/2$ | $h_2/2$ | d_1 | d_2 | J_1 | J_2 | B_{stray}^2 | Energy | OF |
|---------|-------|-------|-----------|---------|-------|-------|----------|----------|---------------|----------|---------|
| | m | m | m | m | m | m | A/mm^2 | A/mm^2 | μT | MJ | - |
| fixed | 2.0 | - | 0.8 | - | 0.27 | - | 22.5 | -22.5 | - | - | - |
| results | - | 3.08 | - | 0.239 | - | 0.394 | - | - | 0.79138 | 180.0277 | 0.08808 |

5.2 8 Parameter Problem, Continuous Case

Best result of the 8 Parameter Problem calculated with (3) and (4) with a strayfield of $B_{norm} = 200 \,\mu\text{T}$:

| | R_1 m | R_2 m | $h_1/2$ m | $h_2/2$ m | d_1 m | d_2 m | J_1 A/mm ² | J_2 A/mm ² | $\begin{array}{c} B_{stray}^2 \\ nT \end{array}$ | Energy MJ | OF - |
|---------------------|---------|---------|-----------|-----------|---------|---------|-------------------------|-------------------------|--|--------------|---------|
| optimal solution | 1.296 | 1.8 | 1.089 | 1.513 | 0.583 | 0.195 | 16.695 | -18.91 | 0.07242 | 180.0 | 0.0018 |

References

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