

# Team Workshop Problem 15

## Rectangular Slot in a Thick Plate: a Problem In Nondestructive Evaluation

### INTRODUCTION

Inversion of eddy-current data and the reconstruction of flaws is the preeminent problem in electromagnetic nondestructive evaluation (NDE). This places a premium on developing good forward models for computing field-flaw interactions, because all inversion algorithms must, of necessity, rely on such calculations. There has evolved in recent years several sophisticated computational models for the forward problem [1-4], but these models differ significantly in their theoretical and numerical approaches. For example, [1-3] use a volume-integral approach that incorporates fast Fourier transforms with conjugate gradients to solve the resulting linear system of equations, whereas [4] uses finite-elements.

Because of this diversity of theoretical-computational approaches, it has become clear that there is now a great need to present experimental data from benchmark problems, whose purpose is to not only validate individual models and codes, but to also allow comparisons between competing models and codes. In this paper we present two such problems for the calculation of impedance change,  $Z$ . These problems,

1. Rectangular slot in a thick plate (900 Hz)
2. Rectangular slot in a thick plate (7 kHz),

have the common feature of being based on practical eddy-current testing techniques, and of utilizing simple geometries.

Additional problems of this genre, including cracks in a thin plate, cracks in a double plate system, and cracks in a thin plate with a tangent coil, are collected together in [6]; further details of each experiment can be found in the references cited in this paper.

### PROBLEM NO. 1

The experimental arrangement is shown schematically in Figure 1. Here, a circular air-cored coil is scanned, parallel to the x-axis, along the length of a rectangular slot in an aluminum alloy plate. Both the frequency and the coil lift-off are fixed and  $Z$  is measured as a function of coil-center position. The parameters for this test experiment are listed in Table 1. This problem is completely described in [7], and is also included in [5]. Solutions appear in [3,8], where a volume-integral equation is used. Preliminary calculations for this case were first reported by Dunbar [9].

### PROBLEM NO. 2

The rectangular slot geometry for this problem is identical to that of Problem No. 1 (see Figure 1). The experimental arrangement uses a larger coil, at a higher frequency (see Table 2 for the parameters). The skin depth at this frequency is one-fifth of the slot depth, which makes this problem differ from No.1 by nearing the thin-skin limit. Theoretical calculations for this problem have been published [8].

## OBJECTIVE

The objective is to compute the change in the inductance and resistance of the driving-point impedance of the coil (compared to its value over an unflawed portion of the plate) as a function of coil position, and compare the computed results to the experimental results tabulated in Table 3. This is to be done for each problem. In addition, the computed and experimental results are to be compared by plotting the magnitude and phase of each versus coil-center position. Plot the magnitude and phase (in degrees) on separate graphs, for each test. The magnitude and phase are given by:

$$\begin{aligned} |Z| &= \sqrt{(X_L)^2 + (R)^2} \\ \text{Arg}(Z) &= \tan^{-1} \frac{X_L}{R}, \end{aligned} \quad (1)$$

where  $X_L = \omega L$ .

## REFERENCES

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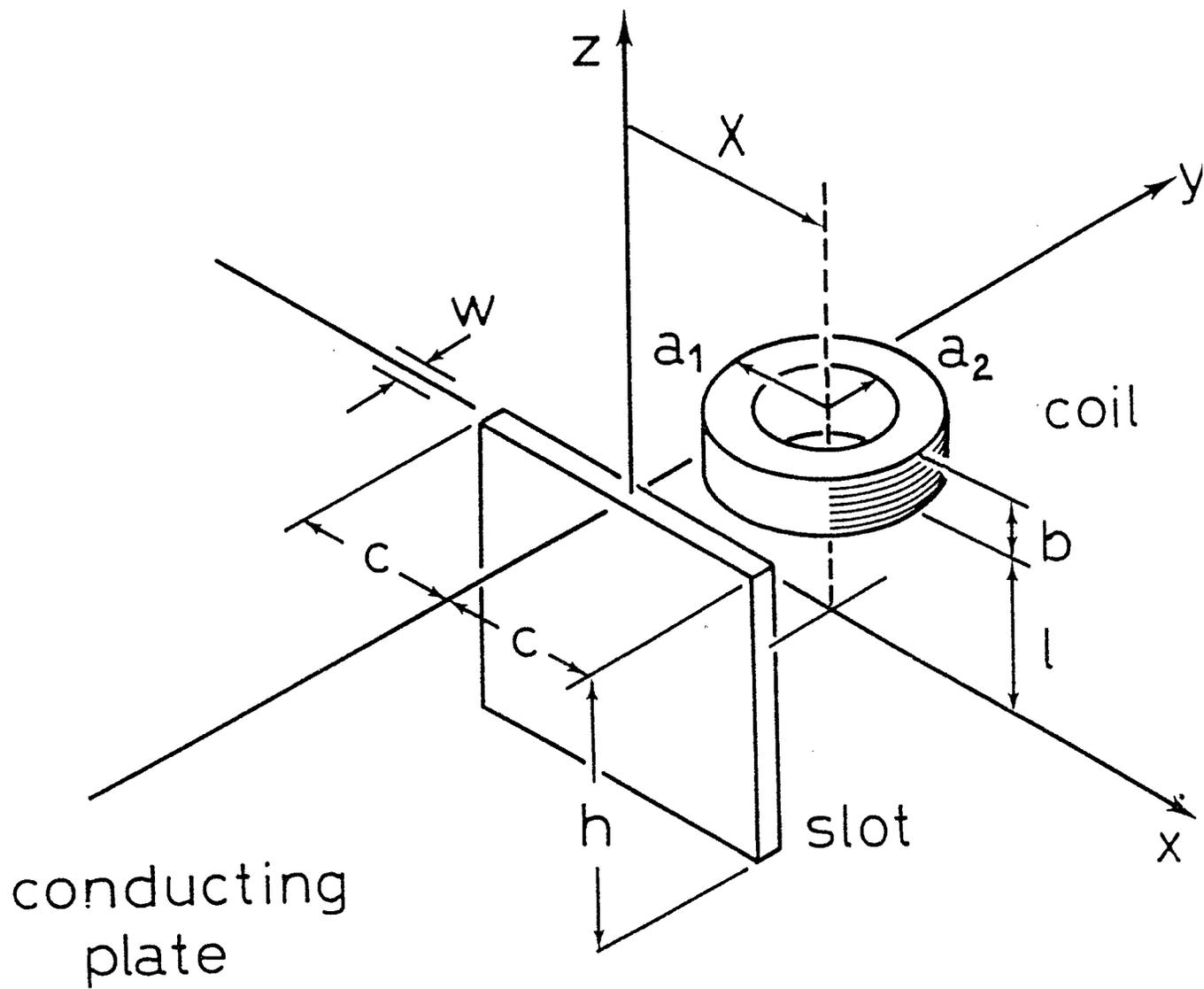


Figure 1: Schematic configuration for the measurement of  $Z$  due to a surface breaking slot.

Table 1: Parameters of Test Experiment No. 1 (see Figure 1)

The coil	
Inner radius ( $a_2$ )	$6.15 \pm 0.05$ mm
Outer radius ( $a_1$ )	$12.4 \pm 0.05$ mm
Length ( $b$ )	$6.15 \pm 0.1$ mm
Number of turns ( $N$ )	3790
Lift-off ( $l$ )	0.88mm
The test specimen	
Conductivity ( $\sigma$ )	$3.06 \pm 0.02 \times 10^7$ S/m
Thickness	$12.22 \pm 0.02$ mm
The defect	
Length ( $2c$ )	$12.60 \pm 0.02$ mm
Depth ( $h$ )	$5.00 \pm 0.05$ mm
Width ( $w$ )	$0.28 \pm 0.01$ mm
Other parameters	
Frequency	900 Hz
Skin depth at 900 Hz	3.04mm
Isolated coil inductance	$221.8 \pm 0.04$ mH

Table 2: Parameters of Test Experiment No. 2 (see Figure 1)

The coil	
Inner radius ( $a_2$ )	$9.34 \pm 0.05$ mm
Outer radius ( $a_1$ )	$18.4 \pm 0.05$ mm
Length ( $b$ )	$9.0 \pm 0.20$ mm
Number of turns ( $N$ )	408
Lift-off ( $l$ )	$2.03 \pm 0.05$ mm
The test specimen	
Conductivity ( $\sigma$ )	$3.06 \pm 0.02 \times 10^7$ S/m
Thickness	$12.22 \pm 0.02$ mm
The defect	
Length ( $2c$ )	$12.60 \pm 0.02$ mm
Depth ( $h$ )	$5.00 \pm 0.05$ mm
Width ( $w$ )	$0.28 \pm 0.01$ mm
Other parameters	
Frequency	7000 Hz
Skin depth at 7000 Hz	1.09mm
Isolated coil inductance	$3.96 \pm 0.1$ mH

Table 3: Experimental Results for Problems 1 and 2

#1			#2		
$X(mm)$	$L(mH)$	$R(^\circ)$	$X(mm)$	$L(mH)$	$R(^\circ)$
0.0	0.39	0.2	0.0	0.5	0.02
0.5	0.40	0.2	1.0	0.2	0.02
1.0	0.45	0.1	2.0	0.5	0.02
1.5	0.53	0.0	3.0	0.7	0.04
2.0	0.63	-0.1	4.0	1.4	0.05
2.5	0.76	-0.2	5.0	2.0	0.06
3.0	0.91	-0.4	6.0	3.0	0.08
3.5	1.06	-0.6	7.0	3.9	0.11
4.0	1.22	-0.8	8.0	5.2	0.13
4.5	1.38	-1.0	9.0	6.6	0.15
5.0	1.55	-1.2	10.0	7.5	0.17
5.5	1.70	-1.35	11.0	8.6	0.18
6.0	1.84	-1.5	12.0	9.3	0.19
6.5	1.96	-1.7	13.0	9.5	0.19
7.0	2.07	-1.8	14.0	9.5	0.19
7.5	2.15	-1.85	15.0	9.1	0.18
8.0	2.22	-1.85	16.0	8.4	0.18
8.5	2.25	-1.9	17.0	7.3	0.16
9.0	2.27	-1.85	18.0	6.4	0.14
9.5	2.26	-1.8	19.0	5.0	0.12
10.0	2.23	-1.65	20.0	4.1	0.10
10.5	2.16	-1.55	21.0	3.0	0.08
11.0	2.08	-1.45	22.0	2.3	0.07
11.5	1.98	-1.3	23.0	1.6	0.05
12.0	1.88	-1.1	24.0	1.1	0.04
12.5	1.75	-0.9	25.0	0.7	0.03
13.0	1.62	-0.6	26.0	0.5	0.02
13.5	1.45	-0.4	27.0	0.5	0.02
14.0	1.31	-0.2	29.0	0.2	0.01
14.5	1.16	-0.1	31.0	0.0	0.00
15.0	1.00	0.1			
15.5	0.86	0.3			
16.0	0.72	0.35			
16.5	0.60	0.5			
17.0	0.49	0.5			
17.5	0.40	0.55			
18.0	0.31	0.55			
18.5	0.25	0.45			
19.0	0.18	0.45			
19.5	0.15	0.3			
20.0	0.11	0.3			
20.5	0.07	0.3			
21.0	0.05	0.2			
22.0	0.03	0.1			

Typical errors in AL are  $\pm 0.04mH$  in #1, and  $\pm 0.02\mu H$  in #2.  
 Typical errors in AR are  $\pm 0.1^\circ$  in both experiments.