# Team Problem 13

# 3-D Non-Linear Magnetostatic Model

## 1. General Description

The model is shown in Fig.1. An exciting coil is set between two steel channels, and a steel plate is inserted between the channels. The coil is excited by dc current. The ampere turns are 1000 and 3000 AT which is sufficient to saturate the steel.
The problem is to calculate magnetic fields at various positions.

## 2. Analyzed Region and Boundary Conditions

If the symmetrical and periodic boundary conditions[1] can be used, the 1/4 region shown in Fig.2(a) is enough to be analyzed. The analysis of 1/2 region shown in Fig.2(b) using only symmetrical boundary condition is also acceptable.

## 3. Mesh Description

The mesh is not specified.

## 4. Nonlinearity

The B-H curve of the steel shown in Fig.3 is to be used. The typical values of B(T) and H(A/m) are also shown in Fig.3. The curve for high flux densities (B>1.8T) should be approximated by Eq.(1):

$$
\begin{aligned}
&B=\mu_0 H + (aH^2 + bH + c) && (1.8 \leq B \leq 2.22\mathrm{T}) \\
&B=\mu_0 H + Ms && (B \geq 2.22\mathrm{T})
\end{aligned}
\qquad (1)
$$

where $\mu_0$ is the permeability of free space. The constants a, b and c are $-2.381\mathrm{x}10^{-10}$, $2.327\mathrm{x}10^{-5}$, and *1.590* respectively. Ms is the saturation magnetization (2.16T) of the steel. Equation (1) shows that the steel part is assumed to be completely saturated when B is higher than 2.22T.

## 5. Quantities and Distributions to be Calculated

### 5a. Points where flux densities are compared

To compare results, please complete Tables 1, 2 and 3. Fig.4 shows the specified positions for average flux density in the steel and flux density in the air. Fig.5 shows the recommended points to be compared. The points ① to ④ are for comparison between various numerical methods of analysis. The points where large errors may occur, such as due to large flux density changes, are chosen. The points ⑤ to ⑧ show the recommended points to be compared with experiment. Around these points, flux densities can be measured accurately because the gradient of flux density is not so high.

## 5b. Distributions of flux density vectors

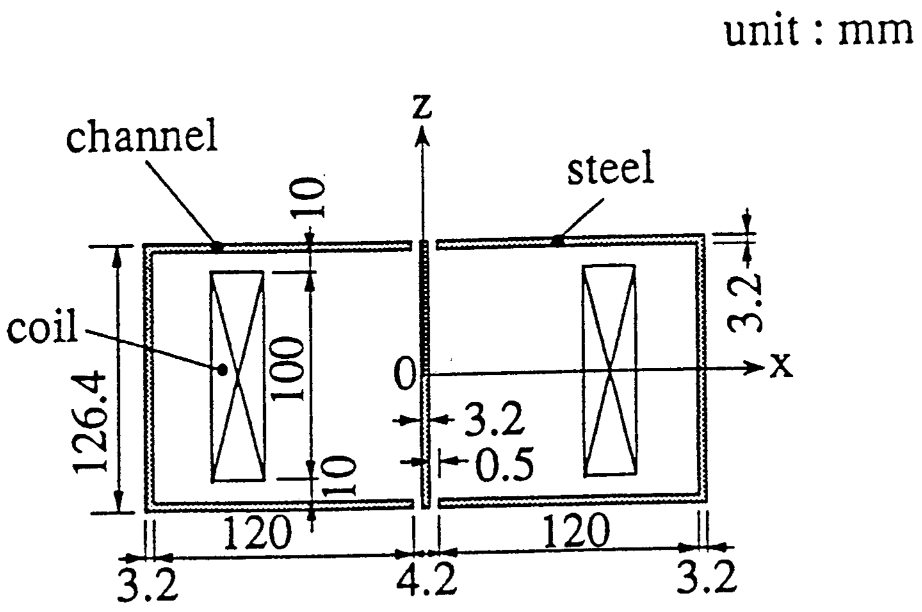
Distributions of flux density vectors on the x-y plane at z=63.2mm, and on the y-z plane at x=0mm are to be presented.

## 6. Description of Computer Program

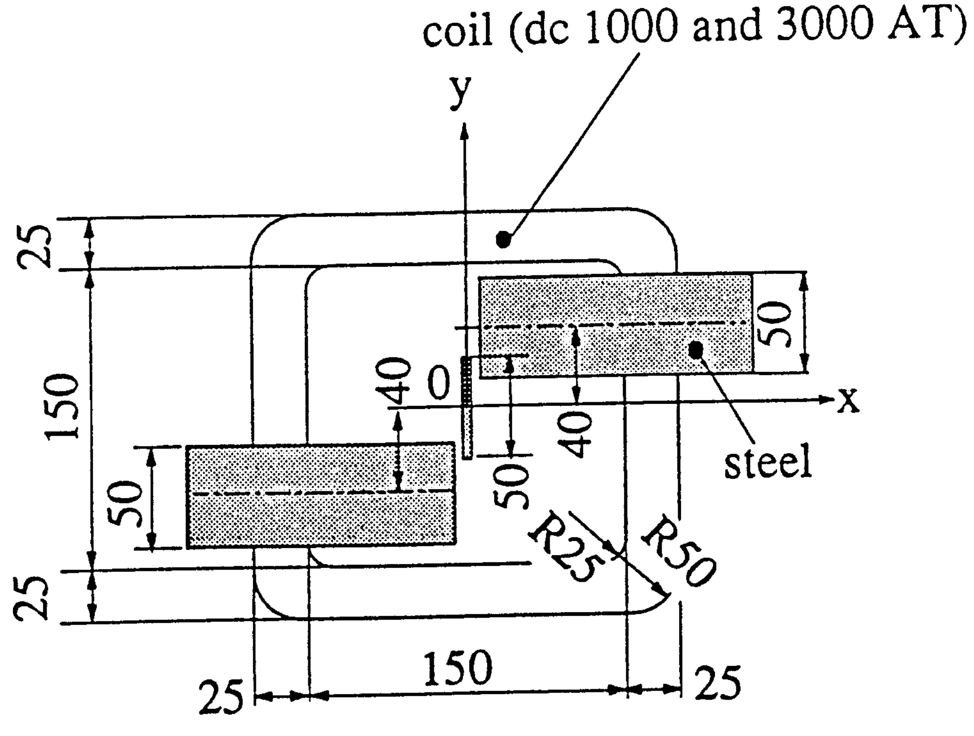
To compare formulations, variables, etc., please complete Table 4. The used memory in the item No.17 in Table 4 is defined as the sum of dimensions declared in the program.

## 7. References

[I] T.Nakata, N.Takahashi, K.Fujiwara & A.Ahagon "Periodic Boundary Condition for 3-D Magnetic Field Analysis and its Applications to Electrical Machines", IEEE Trans. Magnetics, MAG-24, 6, 2694 (1988).
[2) O.C.Zienkiewicz "The Finite Element Method (Third Edition)", McGraw-Hill (1977).
[3] P.P.Silvester, H.S.Cabayan & B.T.Browne: "Efficient Techniques for Finite Element Analysis of Electrical Machines", IEEE Trans: PA&S, PAS-92, 6, 1274 (1973).
[4] J.H.Hwang & W.Load : "Finite Element Analysis of the Magnetic Field Distribution inside a Rotating Ferromagnetic Bar", IEEE Trans. Magnetics, MAG-lO, 4, 1113 (1974).
[5] H.Akima : "A New Method of Interpolation and Smooth Curve Fitting Based on Local Procedures", Journal of ACM, 17, 4, 589 (1970).
[6] C.R.I.Emson "Methods for the Solution of Open-Boundary Electromagnetic-Field Problems", lEE Proc., 135, Pt.A, 3, 151(1988).
[7] P.Tong & J.N.Rossetos : "Finite-Element Method (Basic Technique and Implementation)", MIT Press (1977).
[8] P.Sonneveld "CGS, a Fast Lanczos-Type Solver for Nonsymmetric Linear Systems", Report 84-16, Department of Mathematics and Informatics, Delft University of Technology, The Netherlands (1984).
[9] A.Bossavit & 3.C.Verite : "The "TRIFOU" Code : Solving the 3-D Eddy-Currents Problem by Using H as State Variable", IEEE Trans. Magnetics, MAG-19, 6, 2465 (1983).

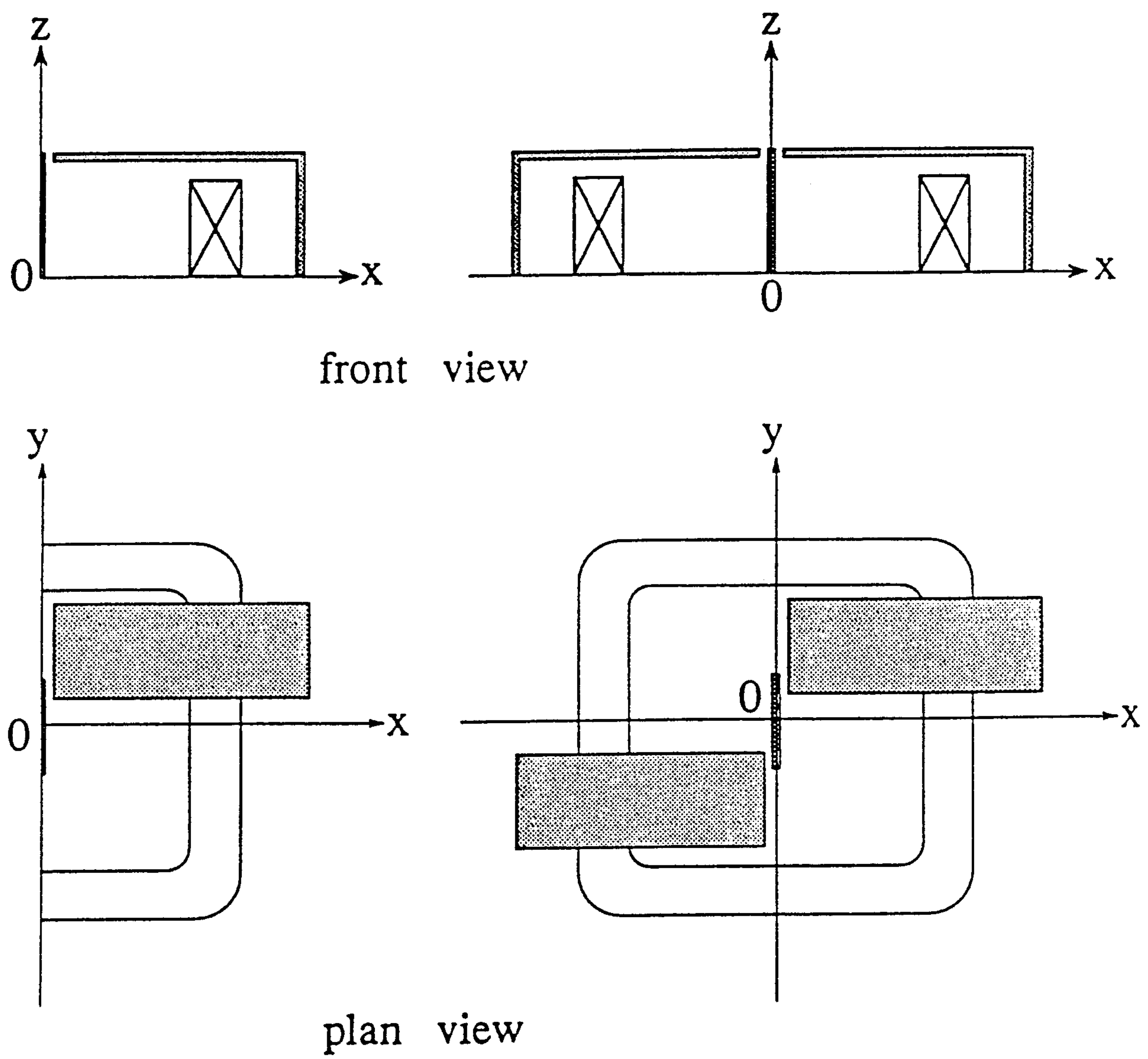


(a) front view



(b) plan view

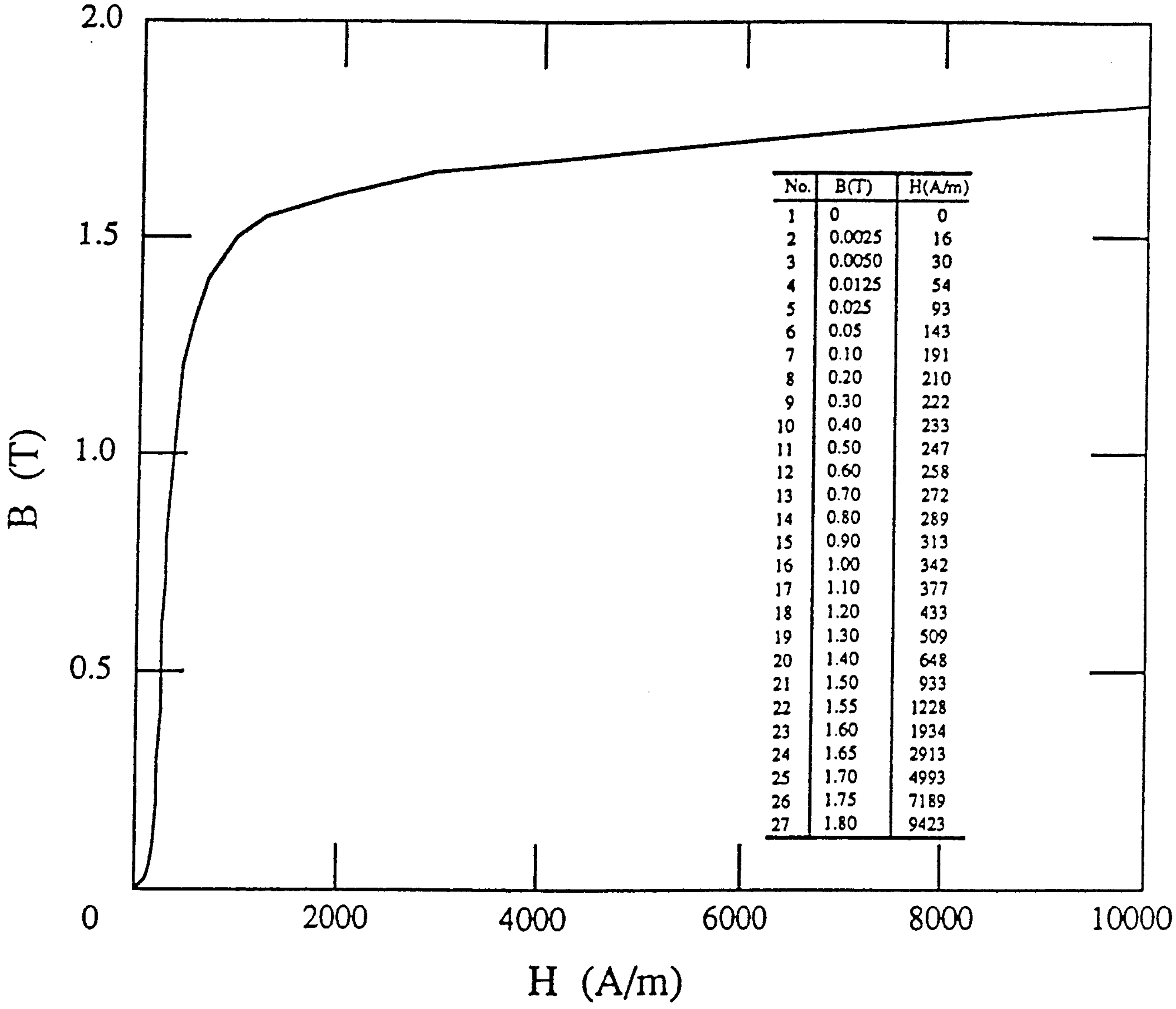
Fig. 1. 3-D nonlinear magnetostatic model



(a) 1/4 region

(b) 1/2 region

Fig. 2. Analyzed region



| No. | B(T) | H(A/m) |
|---|---|---|
| 1 | 0 | 0 |
| 2 | 0.0025 | 16 |
| 3 | 0.0050 | 30 |
| 4 | 0.0125 | 54 |
| 5 | 0.025 | 93 |
| 6 | 0.05 | 143 |
| 7 | 0.10 | 191 |
| 8 | 0.20 | 210 |
| 9 | 0.30 | 222 |
| 10 | 0.40 | 233 |
| 11 | 0.50 | 247 |
| 12 | 0.60 | 258 |
| 13 | 0.70 | 272 |
| 14 | 0.80 | 289 |
| 15 | 0.90 | 313 |
| 16 | 1.00 | 342 |
| 17 | 1.10 | 377 |
| 18 | 1.20 | 433 |
| 19 | 1.30 | 509 |
| 20 | 1.40 | 648 |
| 21 | 1.50 | 933 |
| 22 | 1.55 | 1228 |
| 23 | 1.60 | 1934 |
| 24 | 1.65 | 2913 |
| 25 | 1.70 | 4993 |
| 26 | 1.75 | 7189 |
| 27 | 1.80 | 9423 |

Fig. 3. B-H curve of steel

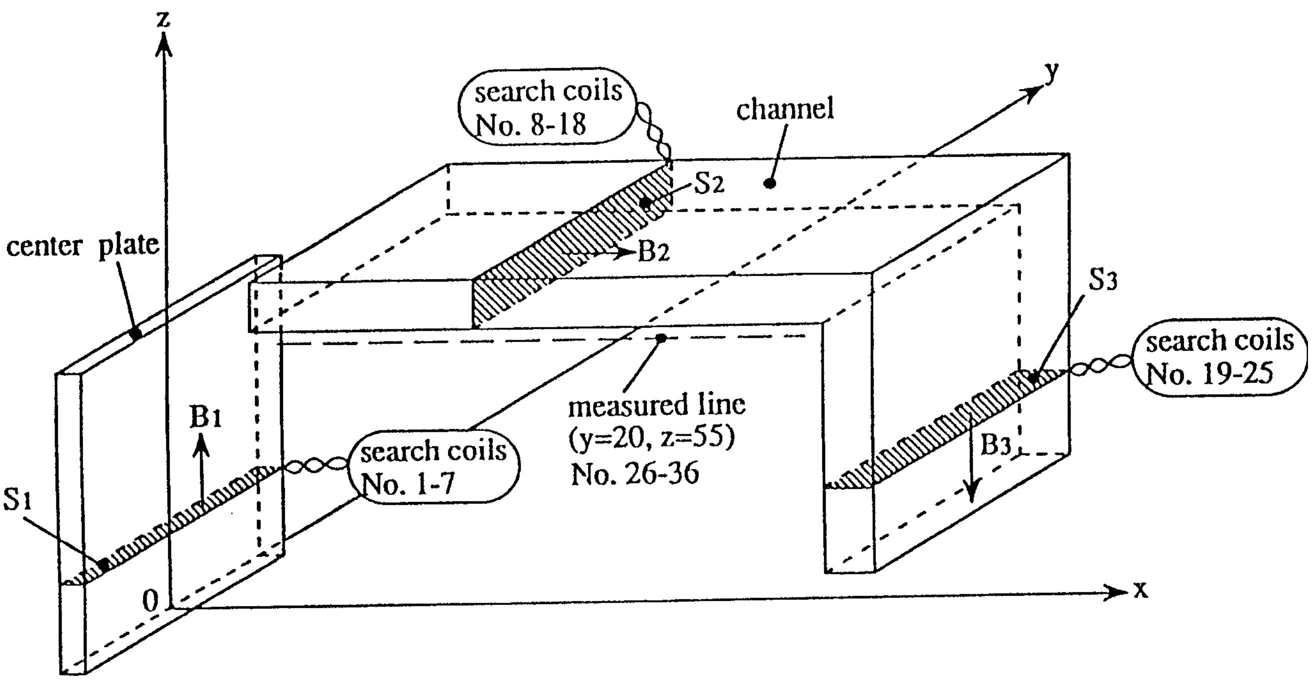
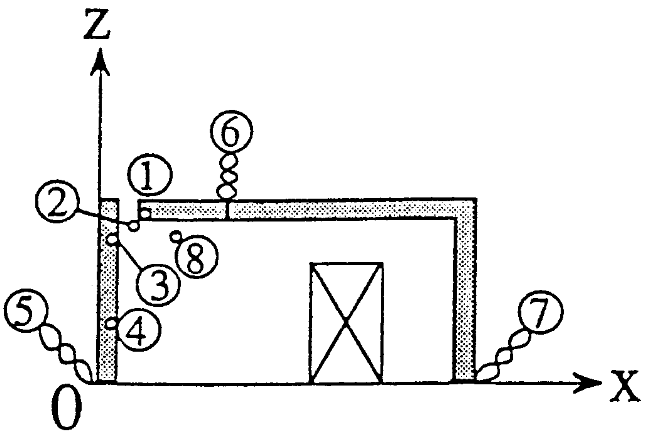
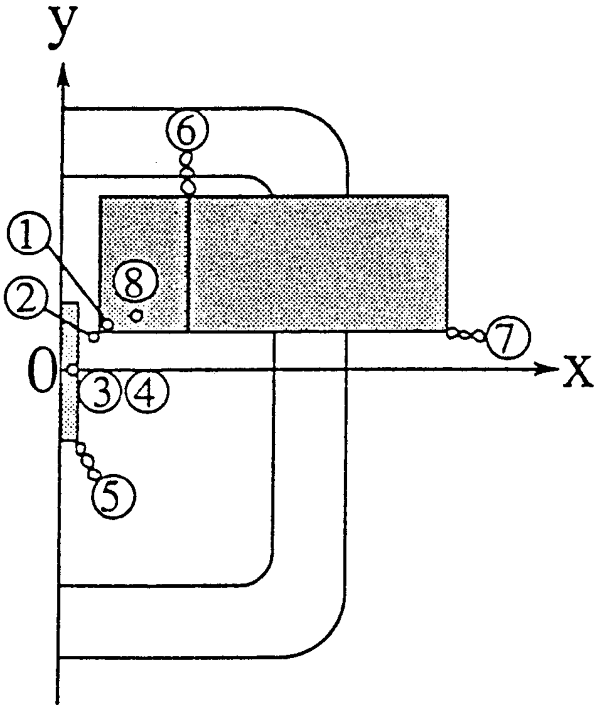


Fig. 4. Specified positions for flux density (see Tables 1 and 2)



(a) front view



(b) plan view

Comparison between various numerical methods :
- the points where the flux densities change suddenly - - - - - ①, ②
- the point where the permeability changes suddenly - - - - - ③
- the point where the error due to the cancellation may be large - - - - - ④

Comparison between calculation and experiment :
- the average flux densities - - - - - ⑤, ⑥, ⑦ (No.1, 12 and 25)
- the point where the flux density is high
  and it does not change suddenly - - - - - ⑧ (No.27)

Fig. 5. Recommended points to be compared (see Table 1, 2, and 3)

# Table 1 Average flux density |B| (T) in the steel (see Fig.4)

| | coordinates (mm) | | | ampere turn (AT) | |
|---|---|---|---|---|---|
| No. | x | y | z | 1000 | 3000 |
| 1 | 0.0≤x≤1.6 | −25.0≤y≤25.0 | 0.0 | | |
| 2 | | | 10.0 | | |
| 3 | | | 20.0 | | |
| 4 | | | 30.0 | | |
| 5 | | | 40.0 | | |
| 6 | | | *50.0* | | |
| 7 | | | 60.0 | | |
| 8 | 2.1 | 15.0≤y≤65.0 | 60.0≤z≤63.2 | | |
| 9 | 10.0 | | | | |
| 10 | 20.0 | | | | |
| 11 | 30.0 | | | | |
| 12 | 40.0 | | | | |
| 13 | 50.0 | | | | |
| 14 | 60.0 | | | | |
| 15 | 80.0 | | | | |
| 16 | 100.0 | | | | |
| 17 | 110.0 | | | | |
| 18 | 122.1 | | | | |
| 19 | 122.1≤x≤125.3 | 15.0≤y≤65.0 | 60.0 | | |
| 20 | | | 50.0 | | |
| 21 | | | 40.0 | | |
| 22 | | | 30.0 | | |
| 23 | | | 20.0 | | |
| 24 | | | 10.0 | | |
| 25 | | | 0.0 | | |

## Table 2 Flux density |B| (T) (see Fig.4)

| No. | coordinates (mm) | | | ampere turn (AT) | |
|---|---|---|---|---|---|
| | x | y | z | 1000 | 3000 |
| 26 | 10.0 | 20.0 | 55.0 | | |
| 27 | 20.0 | | | | |
| 28 | 30.0 | | | | |
| 29 | 40.0 | | | | |
| 30 | 50.0 | | | | |
| 31 | 60.0 | | | | |
| 32 | 70.0 | | | | |
| 33 | 80.0 | | | | |
| 34 | 90.0 | | | | |
| 35 | 100.0 | | | | |
| 36 | 110.0 | | | | |

## Table 3 Flux density |B| (T) (see Fig.5)

| | coordinates (mm) | | | ampere turn (AT) | |
|---|---|---|---|---|---|
| No. | x | y | z | 1000 | 3000 |
| ① | 2.2 | 15.1 | 60.1 | | |
| ② | 2.0 | 14.9 | 50.9 | | |
| ③ | 1.5 | 0.0 | 55.0 | | |
| ④ | 1.5 | 0.0 | 25.0 | | |

to  are for comparison between various numerical methods of analysis. The points where large errors may occur, such as due to large flux density changes, are chosen.

The points ⑤ to ⑧ show the recommended po

# Table 4 Description of computer program

| No. | Item | Specification |
|---|---|---|
| 1 | Code name | |
| 2 | Formulation | ☐ 1. FEM (Finite Element Method)<br>☐ 2. BEM (Boundary Element Method)<br>☐ 3. IEM (Integral Equation Method)<br>☐ 4. FDM (Finite Difference Method)<br>☐ 5. combination ( + )<br>☐ 6. others ( )<br>(Please write references in item No.18) |
| 3 | Governing equations | |
| 4 | Solution variables | |
| 5 | Gauge condition | ☐ 1. not imposed<br>2. imposed<br>☐ (a) impose the condition on governing equations directly<br>☐ (b) penalty function method<br>☐ (c) Lagrange multiplier method<br>☐ (d) others ( )<br>(Please write references in item No.18) |
| 6 | Fraction of geometry | ☐ 1. 1/4 [1]<br>☐ 2. 1/2 |
| 7 | Technique for non-linear problem[2] | ☐ 1 Newton-Raphson method [3]<br>☐ 2. Modified Newton-Raphson-method<br>☐ 3. Incremental method<br>☐ 4. SOR[4]<br>☐ 5. others ( )<br>(Please write references in item No.18) |
| | Convergence criterion | |

# Table 4 Description of computer program (continued)

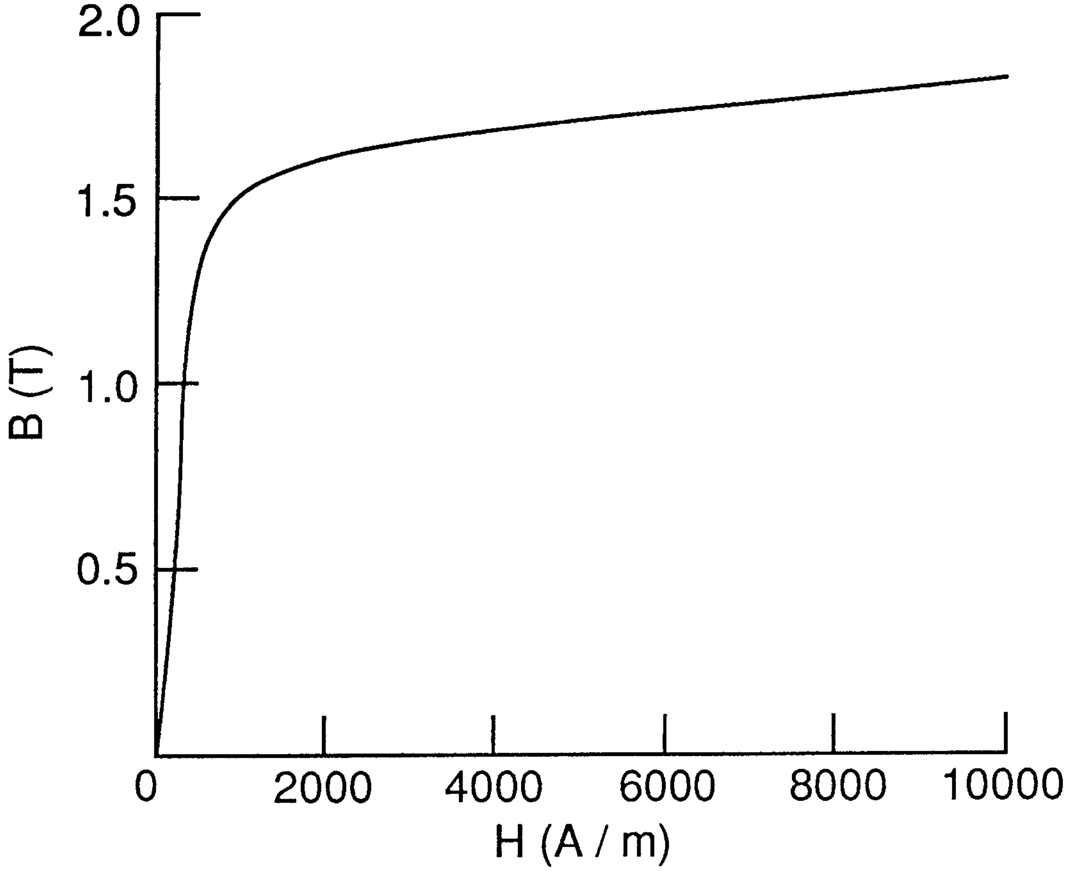
| No. | Item | Specification |
|---|---|---|
| 8 | Approximation method of B-H curve | ☐ 1. spline<br>☐ 2. Akima[5]<br>☐ 3. straight lines<br>☐ 4. others( )<br>(please write references in item No.18) |
| 9 | Technique for open boundary problem [6] | ☐ 1. truncation<br>☐ 2. mapping<br>☐ 3. ballooning<br>☐ 4. Zienkiewicz's infinite element<br>☐ 5. Tong’s infinite element[7]<br>☐ 6. BEM or IEM<br>☐ 7. others ( )<br>(please write references in item No.18) |
| 10 | Calculation method of magnetic field produced by exciting current | ☐ 1. Biot-Savart law (analytical)<br>☐ 2. Biot-Savart law (numerical)<br>☐ 3. taking into account exciting current in governing equations directly |
| 11 | Property of coefficient matrix of linear equations | 1. symmetric<br>☐ (la) sparse<br>☐ (lb) full<br>2. asymmetric<br>☐ (2a) sparse<br>☐ (2b) full<br>☐ 3. combination |
| 12 | Solution method for linear equations | ☐ 1. ICCG<br>☐ 2. ILUBCG<br>☐ 3. ILUCGS[7]<br>☐ 4. SOR<br>☐ 5. $LDL^T$<br>☐ 6. LU<br>☐ 7. Gauss elimination method<br>☐ 8. others ( )<br>(please write references in item No.18) |
|  | Convergence criterion for iteration method |  |

## Table 4 Description of computer program (continued)

| No. | Item | | | Specification | |
|---|---|---|---|---|---|
| 13 | Element type | | | ☐ 1. tetrahedron<br>☐ 2. triangular prism<br>☐ 3. hexahedron<br>☐ 4. triangle<br>☐ 5. rectangle<br>☐ 6. others ( )<br>(please write references in item No.18) | |
| | | | | ☐ 1. nodal element ( nodes )<br>☐ 2. edge element ( edges )[9] | |
| 14 | Number of elements | | | | |
| 15 | Number of nodes | | | | |
| 16 | Number of unknowns | | | | |
| 17 | Computer | name | | | |
| | | speed | | (MIPS), (MFLOPS). | |
| | | main memory (MB) | | | |
| | | used memory (MB) | | | |
| | | precision of data (bits) | | | |
| | | CPU time (sec) | total | | |
| | | | solving linear equations | | |
| 18 | References on Nos.1 to 13, etc. | | | | |

# Average flux density |B| in steel plate (1OOOAT, measured)

| | coordinates (mm) | | | average flux density \|B\|(T) | |
|---|---|---|---|---|---|
| No. | x | y | z | G=0.52(mm) | G=0.47(mm) |
| 1 | 0.0≤x≤1.6 | −25.0≤y≤25.0 | 0.0 | 1.333 | 1.354 |
| 2 | | | 10.0 | 1.329 | 1.339 |
| 3 | | | 20.0 | 1.286 | 1.304 |
| 4 | | | 30.0 | 1.225 | 1.245 |
| 5 | | | 40.0 | 1.129 | 1.138 |
| 6 | | | *50.0* | 0.985 | 0.982 |
| 7 | | | 60.0 | 0.655 | 0.674 |
| 8 | 2.1 | 15.0≤y≤65.0 | 60.0≤z≤63.2 | 0.259 | 0.263 |
| 9 | 10.0 | | | 0.453 | 0.451 |
| 10 | 20.0 | | | 0.554 | 0.563 |
| 11 | 30.0 | | | 0.637 | 0.641 |
| 12 | 40.0 | | | 0.698 | 0.706 |
| 13 | 50.0 | | | 0.755 | 0.763 |
| 14 | 60.0 | | | 0.809 | 0.819 |
| 15 | 80.0 | | | 0.901 | 0.907 |
| 16 | 100.0 | | | 0.945 | 0.958 |
| 17 | 110.0 | | | 0.954 | 0.968 |
| 18 | 122.1 | | | 0.956 | 0.968 |
| 19 | 122.1≤x≤125.3 | 15.0≤y≤65.0 | 60.0 | 0.960 | 0.971 |
| 20 | | | 50.0 | 0.965 | 0.973 |
| 21 | | | 40.0 | 0.970 | 0.982 |
| 22 | | | 30.0 | 0.974 | 0.985 |
| 23 | | | 20.0 | 0.981 | 0.991 |
| 24 | | | 10.0 | 0.984 | 0.995 |
| 25 | | | 0.0 | 0.985 | 0.995 |



The curve for high flux densities (B > 1.8T) is approximated as follows:

$\mu_o H + (aH^2 + bH + c)$ $(1.8T \leq B \leq 2.22T)$

$\mu_o H + Ms$ $(B \geq 2.22T)$

$\mu_o$ : permeability of free space

a : $-2.822 \times 10^{-10}$

b : $2.529 \times 10^{-5}$

c : 1.591

Ms : saturation magnetization (2.16T)

B-H curve of steel.

### Typewritten data for the B-H curve

| No. | B (T) | H (A / m) |
|---|---|---|
| 1 | 0 | 0 |
| 2 | 0.025 | 45 |
| 3 | 0.05 | 75 |
| 4 | 0.10 | 120 |
| 5 | 0.20 | 173 |
| 6 | 0.30 | 201 |
| 7 | 0.40 | 222 |
| 8 | 0.50 | 240 |
| 9 | 0.60 | 250 |
| 10 | 0.70 | 265 |
| 11 | 0.80 | 280 |
| 12 | 0.90 | 300 |
| 13 | 1.00 | 330 |
| 14 | 1.10 | 365 |
| 15 | 1.20 | 415 |
| 16 | 1.30 | 500 |
| 17 | 1.40 | 640 |
| 18 | 1.50 | 890 |
| 19 | 1.55 | 1150 |
| 20 | 1.60 | 1940 |
| 21 | 1.65 | 3100 |
| 22 | 1.70 | 4370 |
| 23 | 1.75 | 6347 |
| 24 | 1.80 | 8655 |